Lecture *Hierarchical Planning*

Chapter:
Search

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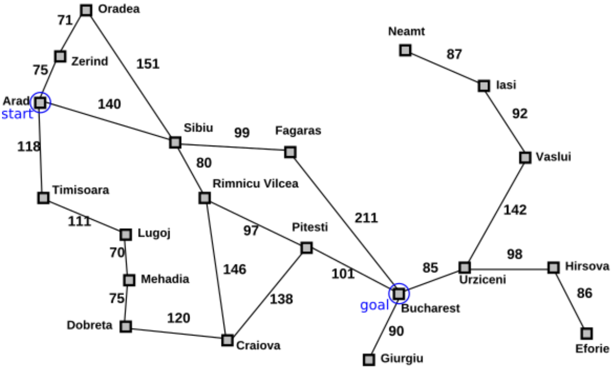
ulm university universität uulm

Overview:

Example



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How to – automatically – find a(n optimal/good) way from *Arad* to *Bucharest?*

(The edge numbers indicate action/traversal costs.)

Introduction

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  - Also, don't confuse the transition system's state with a search node. Search nodes can contain more information, like the path (sequence of transitions) discovered to reach the respective node.
  - Depending on which problem to solve, we might need to rely upon infinite state transition systems (e.g., for POCL and for hierarchical problems).

Introduction, cont'd

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More examples

- Pathfinding (cf. first slide)

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- Anything that can be described with a transition system, e.g., *Cannibals and Missionaries*, ...

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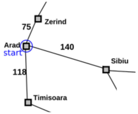

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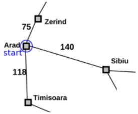

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  - Works also if no heuristics are known.
  - Often very inefficient compared to informed search.

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  - Requires a *visited list* (also called *closed list* or *explored set*).
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  - We need to be more careful if we want to guarantee optimal solutions.

Tree Search vs. Graph Search

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

---

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
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  loop do
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An informal description of the general tree-search and graph-search algorithms. The parts of GRAPH-SEARCH marked in bold italic are the additions needed to handle repeated states.

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- g^* The cost of an optimal solution.

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Correctness If a solution is returned, it is in fact a solution and if "no solution exists" is returned, there does in fact not exist a solution.

BFS Algorithm

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
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Breadth-first search on a graph.

Question:

In which way does this algorithm differ (e.g., is more precise) than the generic graph-search algorithm?

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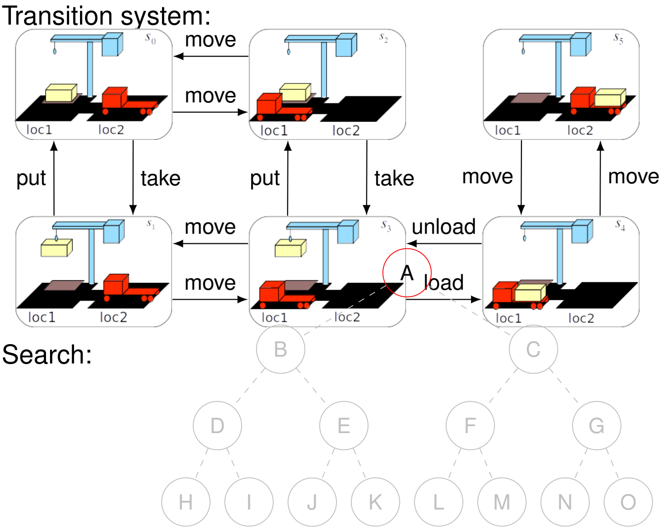
In which way does this algorithm differ (e.g., is more precise) than the generic graph-search algorithm?

- Here, the goal test is done *before insertion into the fringe* ("early goal test"), not after selection from the fringe.
- The fringe is not "generic", but implemented as FIFO.

Example of BFS



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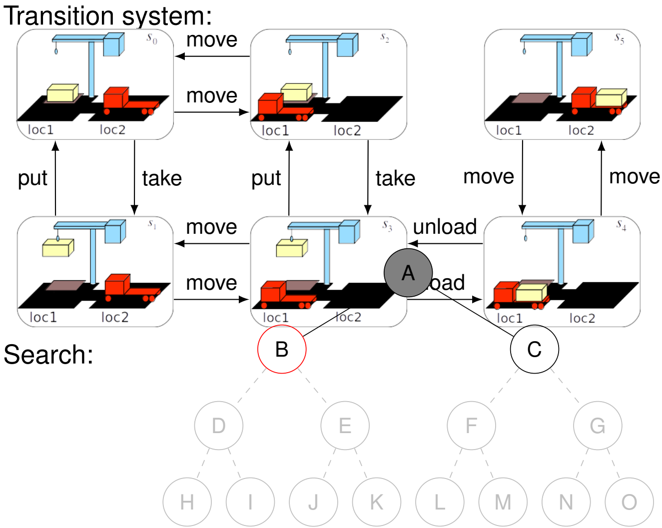

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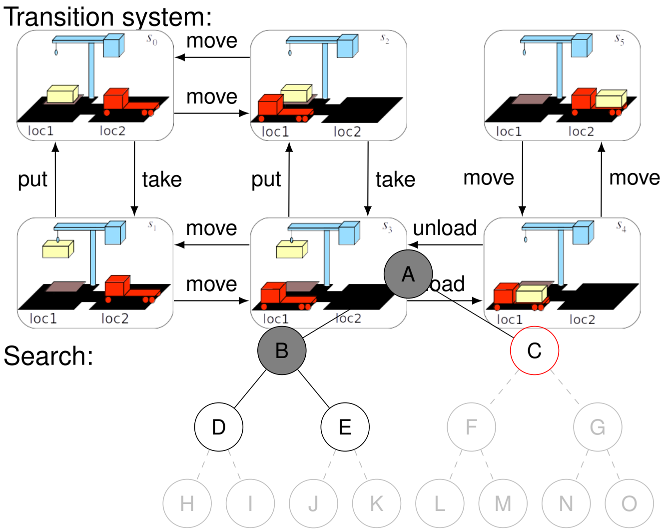

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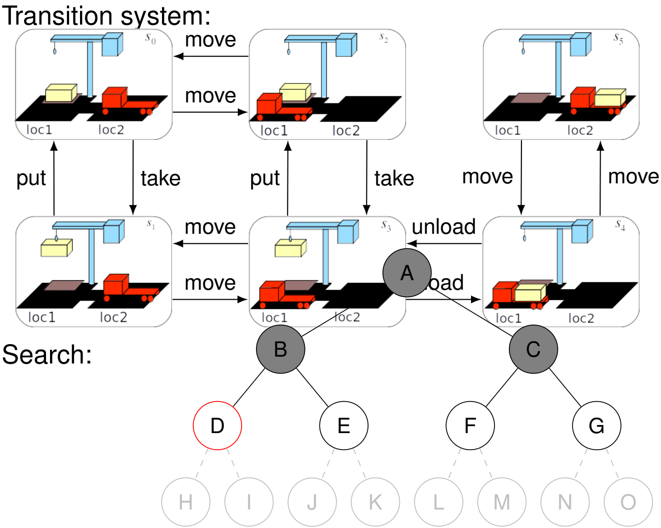

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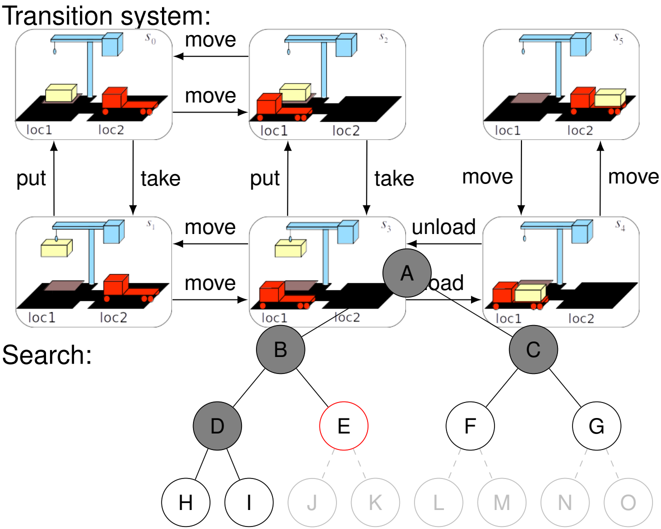

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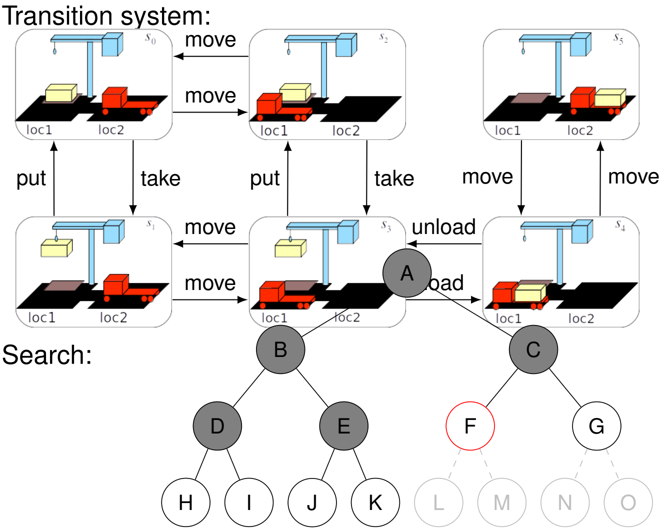

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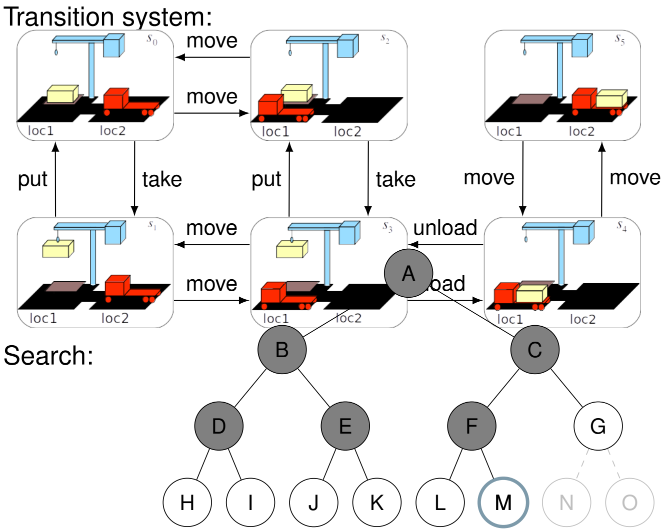
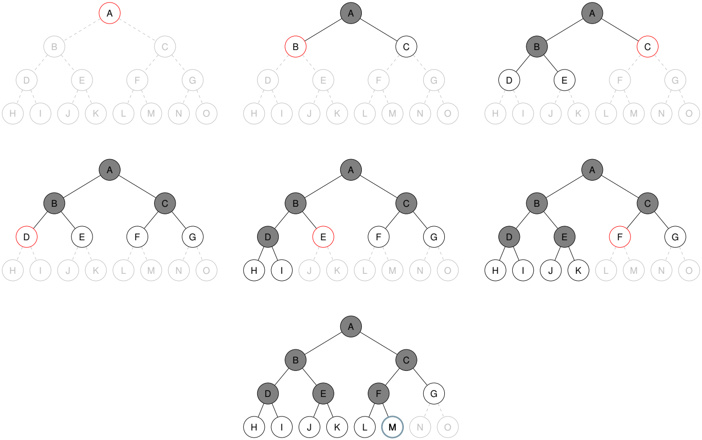


Example of BFS, cont'd

At one glance:



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Space $O(b^d)$

Time

Properties of BFS

Optimality Depends on the costs:

- Without action costs or with unit costs:
  - Obviously (both for tree- and for graph search)
  - Even optimal with an "early goal test"
- With action costs: No

Completeness Depends on properties of the transition system:

- Finite: Yes
- Infinite: Only with finite branching factor.

Correctness Yes

Space $O(b^d)$

Time Just as space. (If the fringe is implemented as a priority queue rather than as queue (FIFO), the queue sorting overhead needs to be added. This does not change the asymptotic runtime, however.)

DFS Algorithm

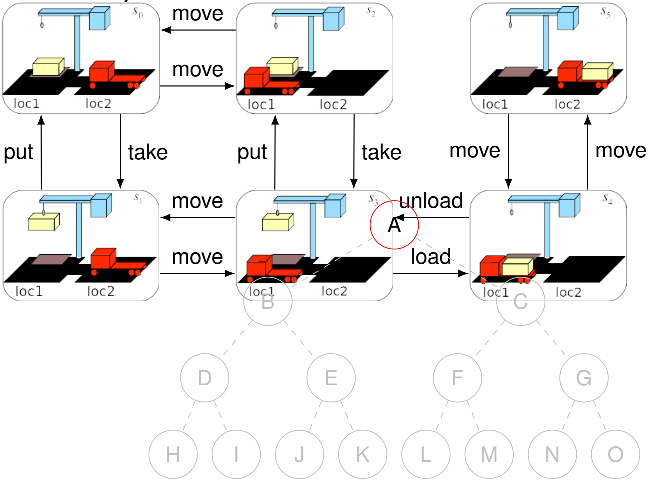
```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

Breadth-first search on a graph.

Just replace the FIFO fringe by a LIFO fringe.

Example of DFS

Transition system:



Search:

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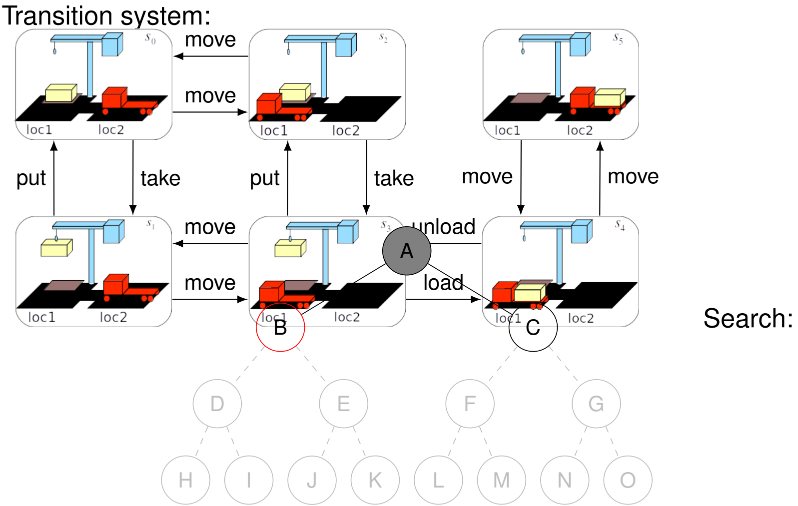

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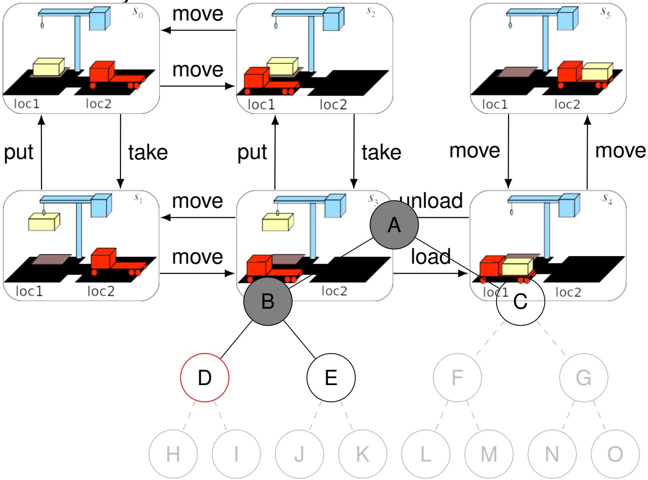

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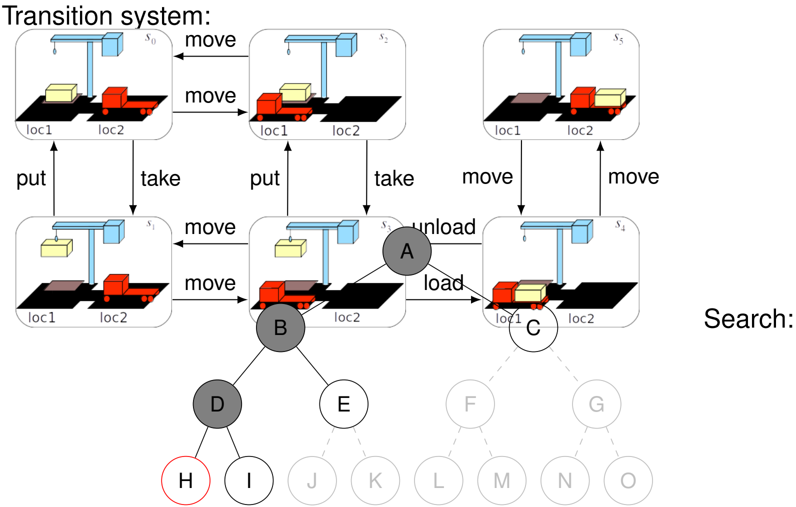

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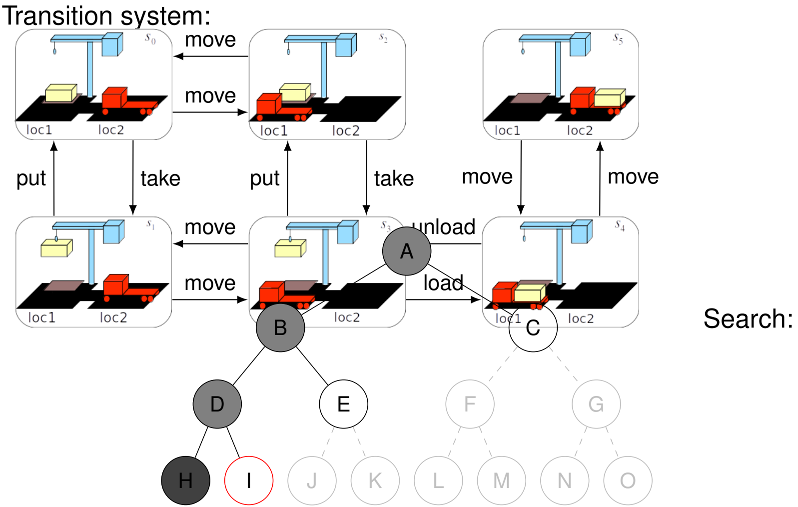

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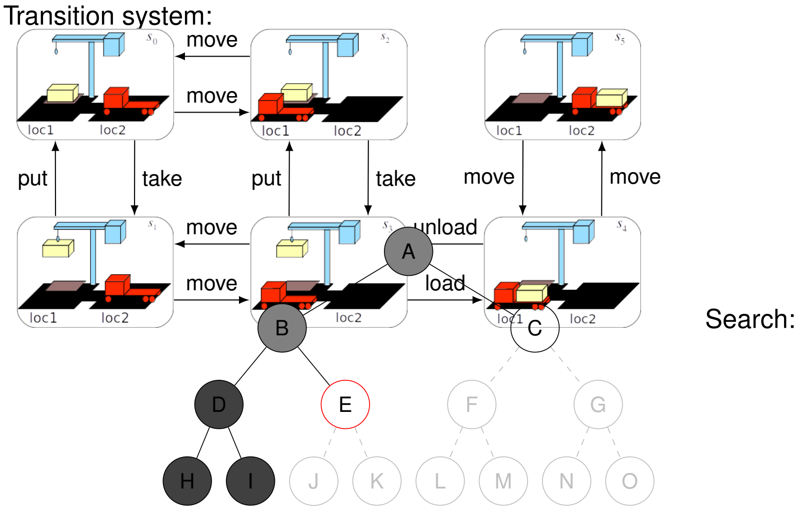

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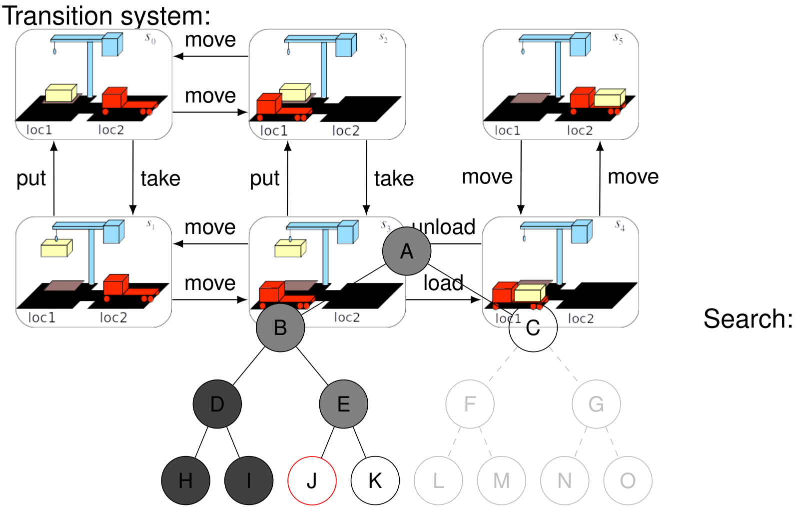

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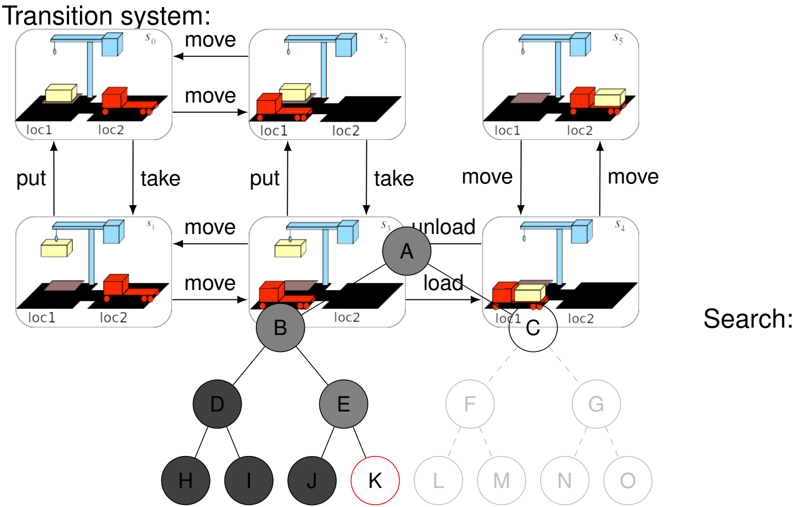

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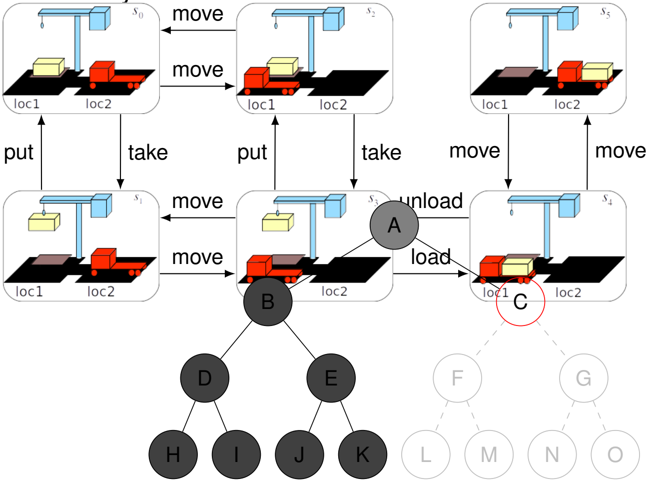

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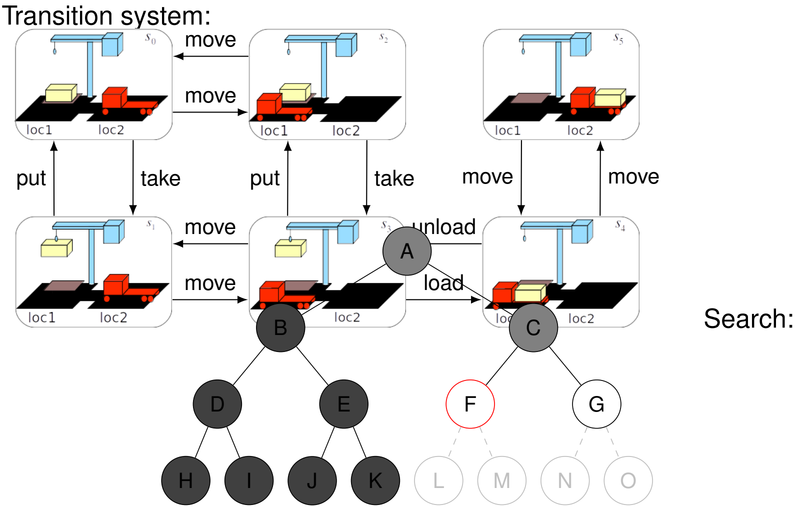

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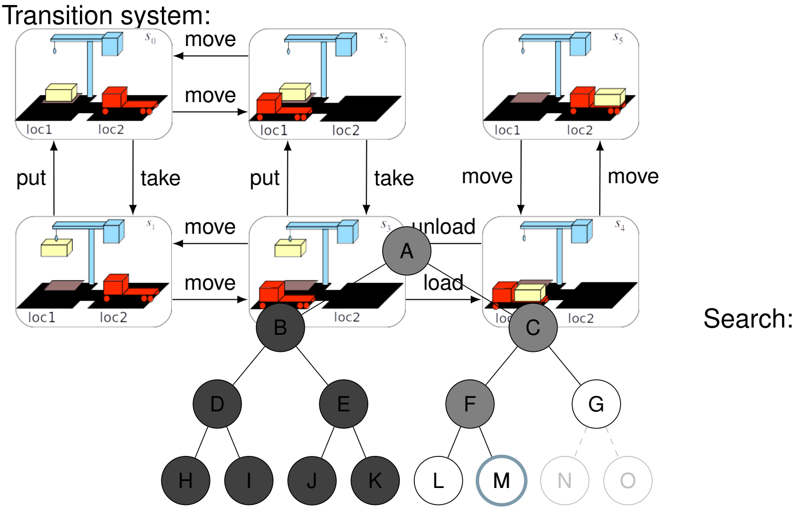
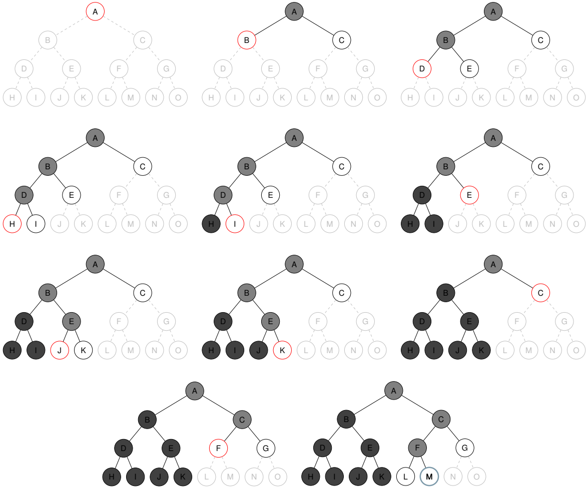


Example of DFS, cont'd

At one glance:



Properties of DFS

Optimality

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Completeness

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- UCS is equivalent to Dijkstra's algorithm.

Properties of Uniform Cost

Optimality

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Completeness

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Time Similar to space.

Motivation

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- What are heuristics? Where do they come from?
- Are there *formal properties* of heuristics?
- How to use/integrate the heuristic? (This forms the algorithm.)

What are Heuristics?

Heuriskein (Greek): Find, Discover

1957
: Methods to identify problem-solving techniques, especially in the field of mathematical proofs.

1963
: Problem-solving processes that potentially deliver solutions.

1971
: "Rules" that domain experts apply in order to find good solutions.

At Present
: Techniques that improve the average performance of problem-solving methods, but not necessarily the worst-case performance.

In Search
: In the context of search methods: functions that estimate solution costs or the goal distance.

Sliding Tile Puzzle

2	1	4	8
9	7	11	10
6	5	15	3
13	14	12	

Problem

→

1	2	3	4
5	6	7	8
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Solution

How far are we still away?

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How far are we still away?

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- “Distance” (horizontal and vertical distance) per tile to goal position $\rightarrow$ *Manhatten distance*

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$\longrightarrow$

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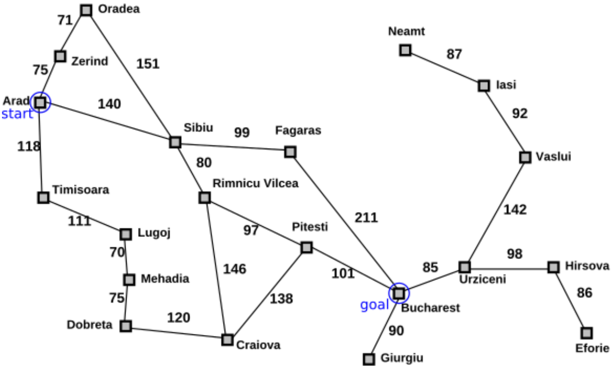
Solution

How far are we still away?

- Number of misplaced tiles
- "Distance" (horizontal and vertical distance) per tile to goal position → *Manhatten distance*
- Ignore certain tiles and use resulting solution cost as estimate.

Road Map

How to find a(n optimal/good) way from *Arad* to *Bucharest*?



copyright: see slide 46[1] (modified)

Possible heuristics?

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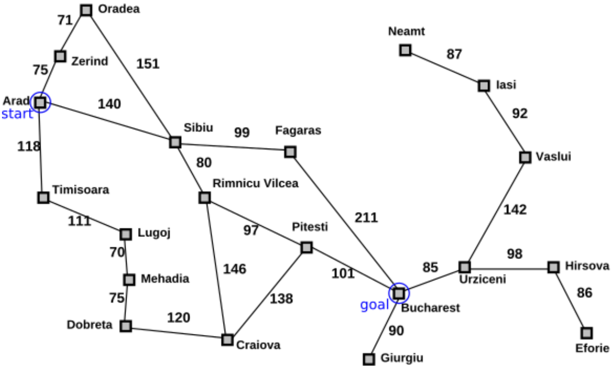
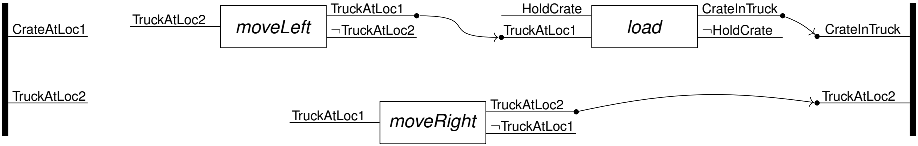


copyright: see slide 46[1] (modified)

Possible heuristics?

- Use the linear distance.

POCL Planning Problem



How many modifications do we have to perform?

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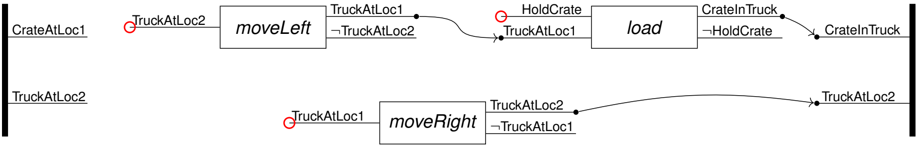

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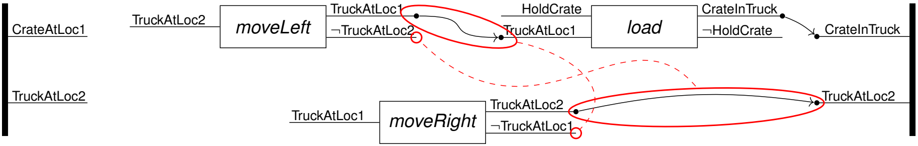

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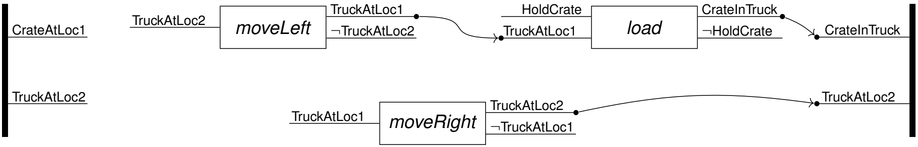

How many modifications do we have to perform?

- Number of open preconditions
- Number of causal treats
- There are a few POCL planning heuristics (for estimating the number of missing modifications or missing actions).

Heuristic Construction

How to come up with heuristics in a *domain-independent* way?

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In *planning*, we can exploit the underlying formalism to design a large set of domain-independent heuristics.

Definitions

Definition (Heuristic, Dominance)

Given a state transition system $ts = (S, L, c, T, I, G)$, a *heuristic h* is a function $h : S \to \mathbb{R}^+ \cup \{\infty\}$. A heuristic h_1 is said to dominate another heuristic h_2 if for all states $s \in S$, $h_1(s) \geq h_2(s)$.

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Heuristics can estimate different metrics (cf. also last lecture). Most common ones are:

- Number of actions of a solution.
- Costs of a solution.
- *Note:* The states of a transition system are not necessarily the same as the states of a planning problem! This depends on the search procedure and the problem class.

Definitions, cont'd I

Definition (Perfect Heuristic)

A heuristic $h^* : S \to \mathbb{R}^+$ is called *perfect*, if for all states $s \in S$ $h^*(s)$ is the cost of the cheapest transition from s to a goal $s' \in G$. Further, $h^*(s) = \infty$ for all states s for which no goal state can be reached.

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Definition (Goal-aware Heuristic)

A heuristic h is called *goal-aware*, if all goal states, i.e., $s_G \in G$ holds $h(s_G) = 0$.

Definitions, cont'd II

Definition (Admissible Heuristics)

A heuristic h is called *admissible*, if for all states $s \in S$, it holds $h(s) \leq h^*(s)$.

Explanation:

Admissible heuristics give a lower (i.e., non-overestimating) bound on the "best" goal distance.

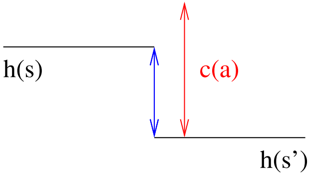
Definitions, cont'd III

Definition (Consistent Heuristics)

A heuristic h is called *consistent*, if for all transitions $(s, l, s') \in T$ holds $h(s) - h(s') \leq c(l)$ (equiv.: $h(s) \leq c(l) + h(s')$).

Explanation:

Consistency: When applying an action *a*, the heuristic value cannot decrease by more than the cost of *a*.



How to Use the Heuristic During Search?

For selecting a search node from the search fringe, pick a search node n with the cheapest f value. f can depend on many properties, for example:

- The heuristic value $h(n)$ of n.
- The cost value $g(n)$ of n.
- The depth $d(n)$ of n.

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Informed Search Algorithms

The most basic informed search algorithms are:

- **Greedy** $f(n) = h(n)$ (also called *Greedy Best-first*)
- **A^*** $f(n) = g(n) + h(n)$
- **Greedy A^*** $f(n) = c(n) + w * h(n), w > 1$ (also called *Weighted A^**)

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- A^* tries to find a cost-minimal solution while taking the heuristic into account during search.
- Greedy A^* is a “more greedy” version of A^*, taking into account the heuristic to a larger extent.

The Search Problem

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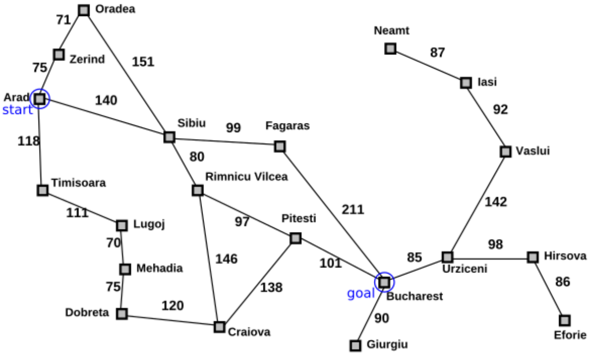

Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

The Search Progress

Reminder:

- Always select a node with minimal $f(n) = h(n)$.
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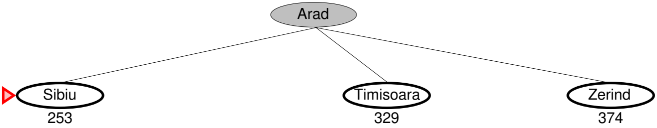
The Search Progress



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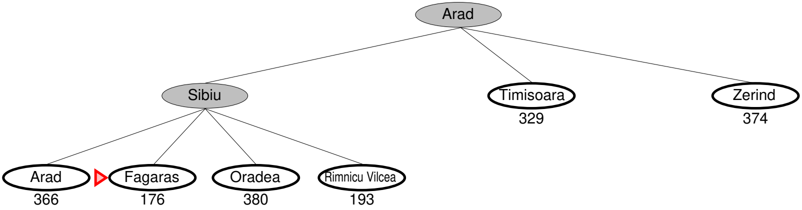

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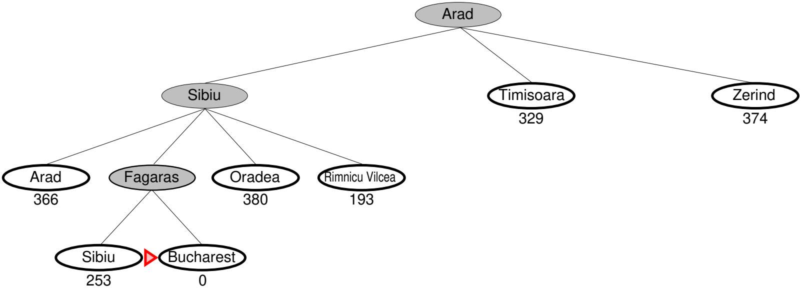

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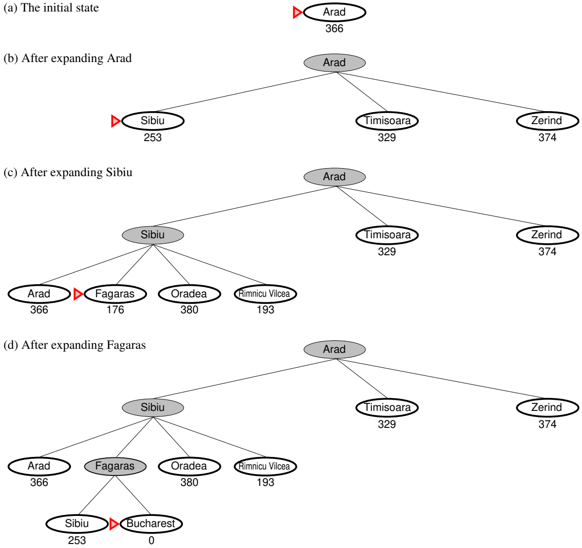

Properties of Greedy Search

Optimality

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Completeness

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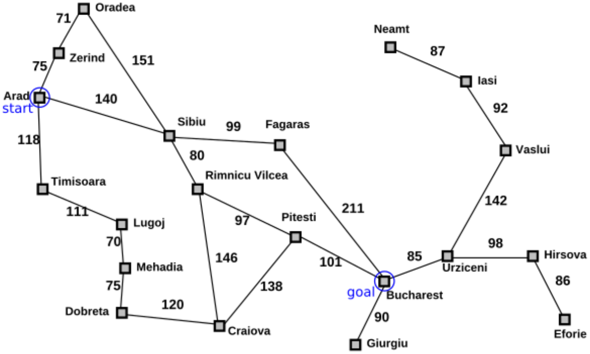
The Search Problem

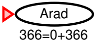
How to find a(n optimal/good) way from *Arad* to *Bucharest*?



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

The Search Progress



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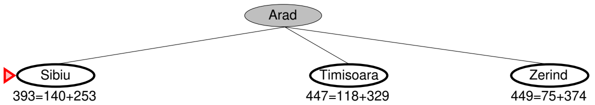
The Search Progress



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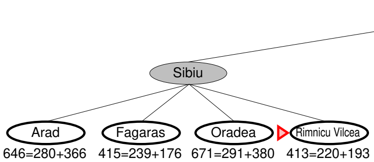

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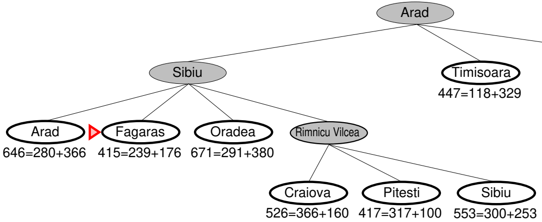

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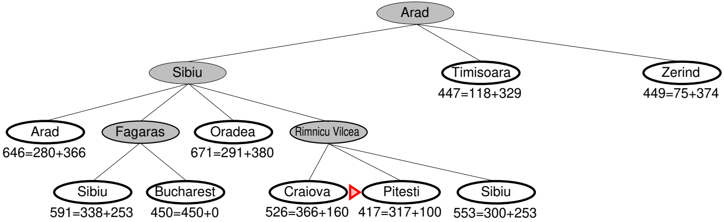

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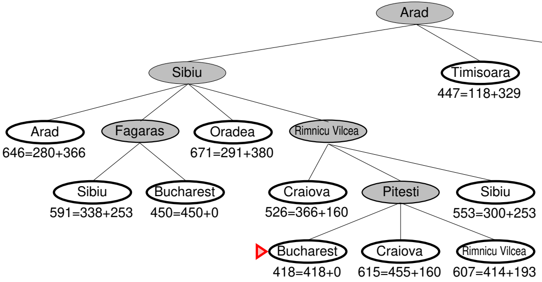

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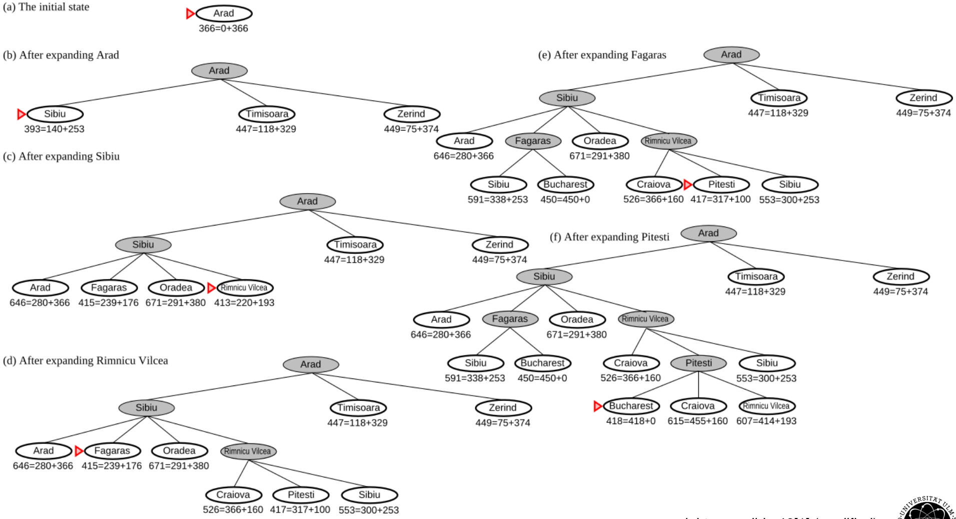

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[1] Title: *Artificial Intelligence: A Modern Approach (Third Edition)*
Url: `https://aima.cs.berkeley.edu/`
Authors: *Stuart Russel* and *Peter Norvig*