Lecture *Hierarchical Planning*

Chapter:
Solving (Non-Hierarchical) Planning Problems via SAT

Gregor Behnke

Institute of Artificial Intelligence,
Ulm University, Germany

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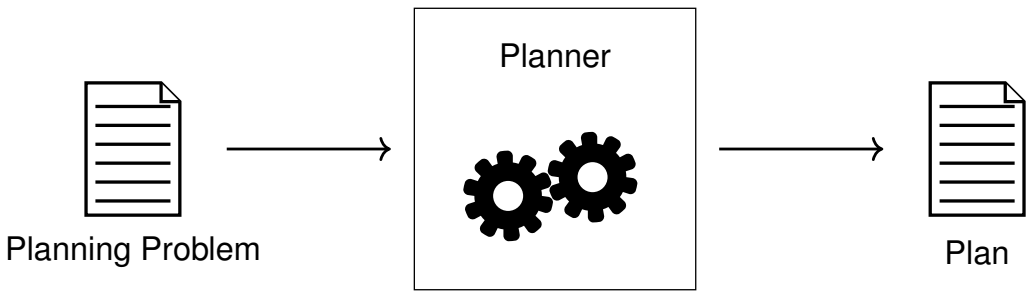
ulm university universität ulm

Overview:

Idea: Problem Transformation



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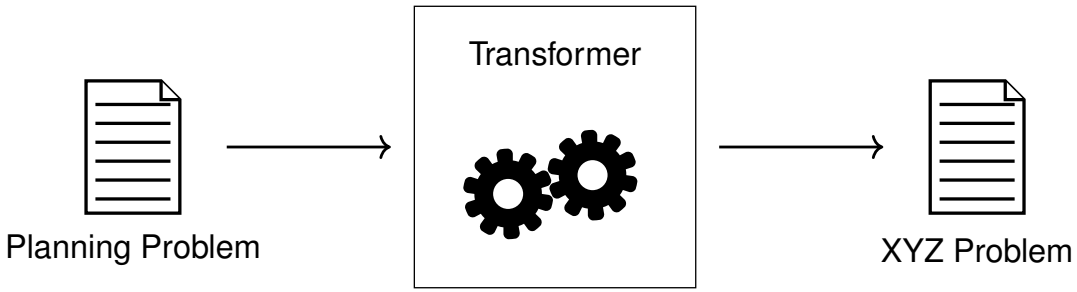

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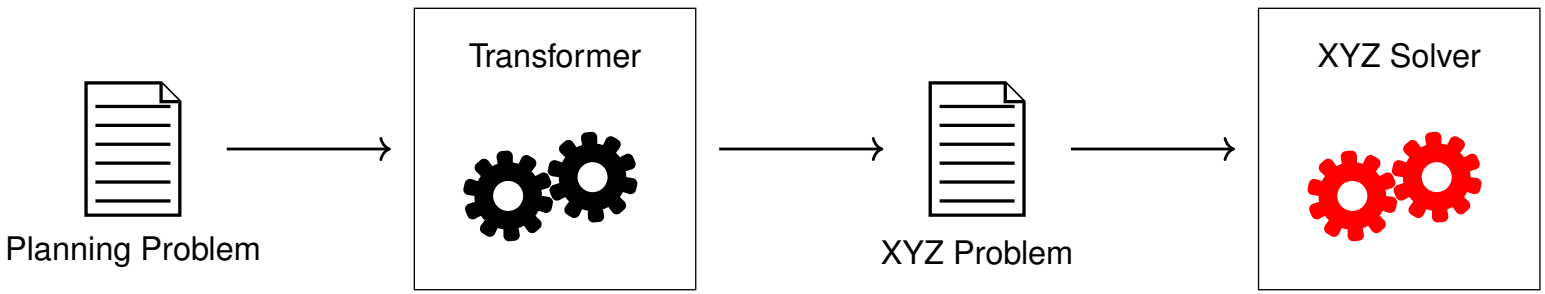

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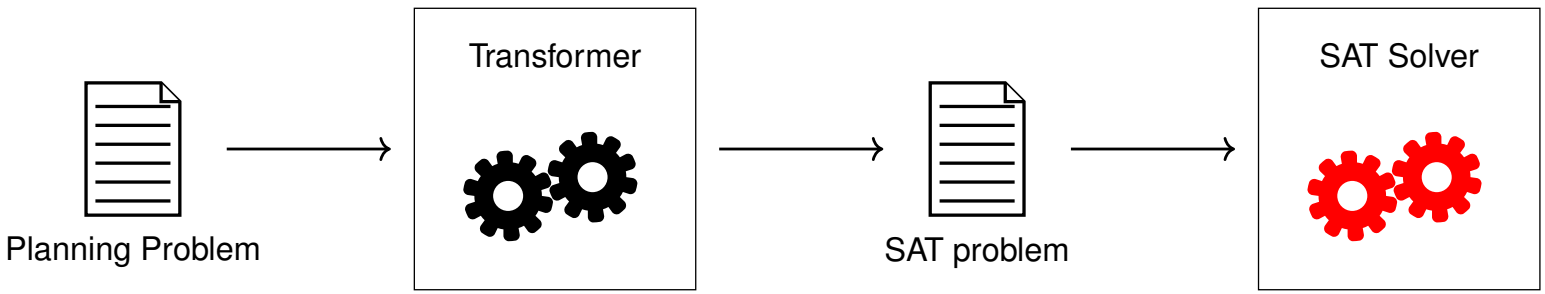

SAT

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Given a propositional formula $\mathcal{F}$, decide whether $\mathcal{F}$ has a satisfying valuation.

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CNF:

$$\mathcal{F} = \bigwedge_{C \in \mathfrak{C}} \bigvee_{\ell \in C} \ell$$

($\mathfrak{C}$ is the set of clauses; C is a clause, a set of literals.)

SAT Solvers

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- Standardised input format DIMACS:

```
p cnf 5 3
1 -5 4 0
-1 5 3 4 0
-3 -4 0
```

$\equiv$

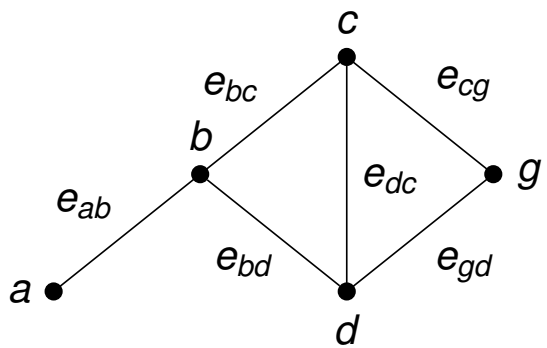
CNF with 5 vars and 3 clauses:

$$(v_1 \vee \neg v_5 \vee v_4) \wedge$$
$$(\neg v_1 \vee v_5 \vee v_3 \vee v_4) \wedge$$
$$(\neg v_3 \vee \neg v_4)$$

Colouring

Definition

Given a graph $G = (V, E)$ and a number k.
Is there an assignment of k colours to the vertices of G, such that all adjacent vertices have different colours?



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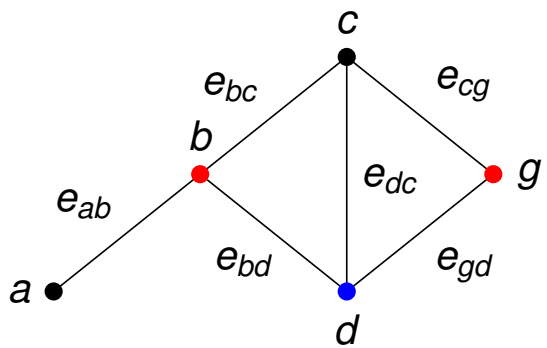

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Variables for choosing the colour of each node

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Every node has at most one colour

$$\bigwedge_{i=1}^{k} \left[\texttt{colour}_v^i \rightarrow \bigwedge_{j=1, i \neq j}^{k} \neg\texttt{colour}_v^j \right] \qquad \forall v \in V$$

Computational Complexity

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Given a planning problem $\mathcal{P}$.
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$\mathbb{PSPACE}$ with $\mathbb{NP}$ calculus?

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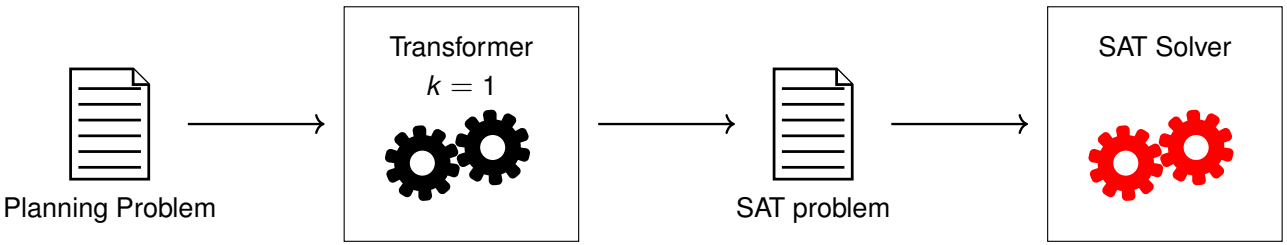
- For full PLANEX: how to choose the plan length?
  - Theoretical limit: $2^{|V|}$.
  - Practical limit: usually smaller (sometimes polynomially bounded).
- Start with a small k and increase until a solution is found.

Bound Iteration



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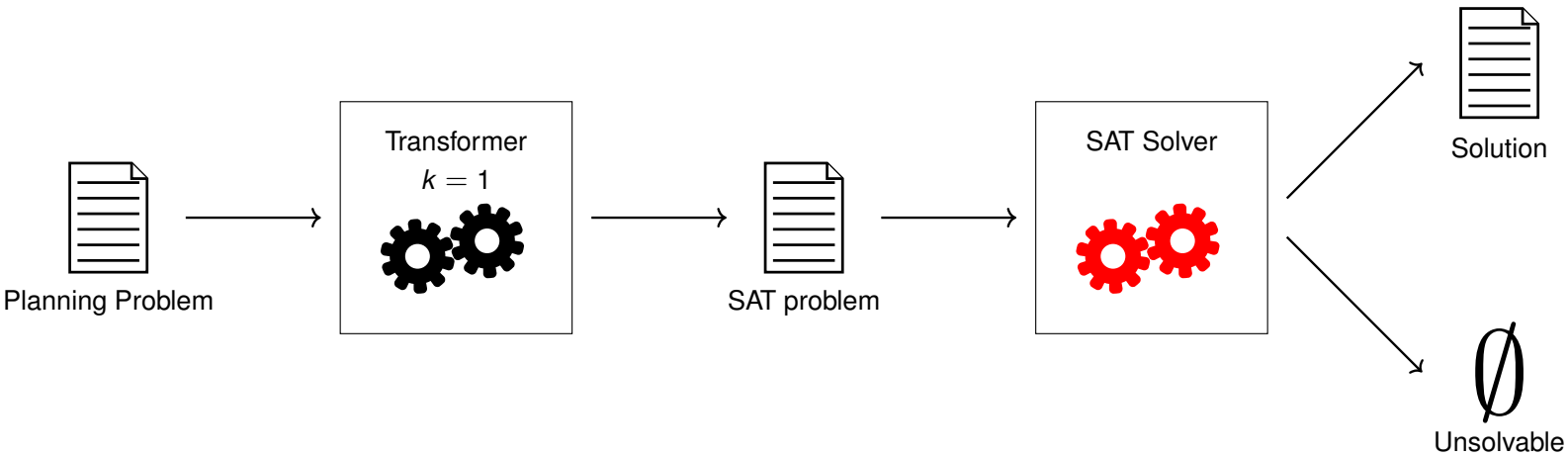

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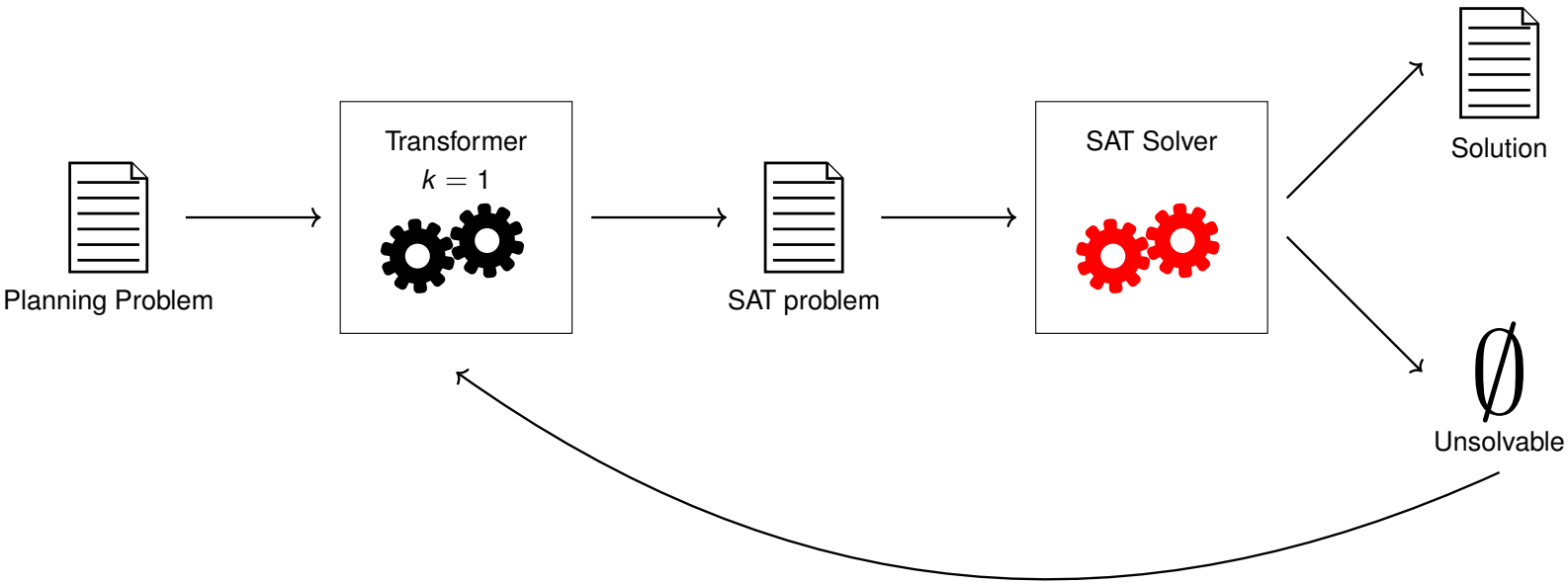

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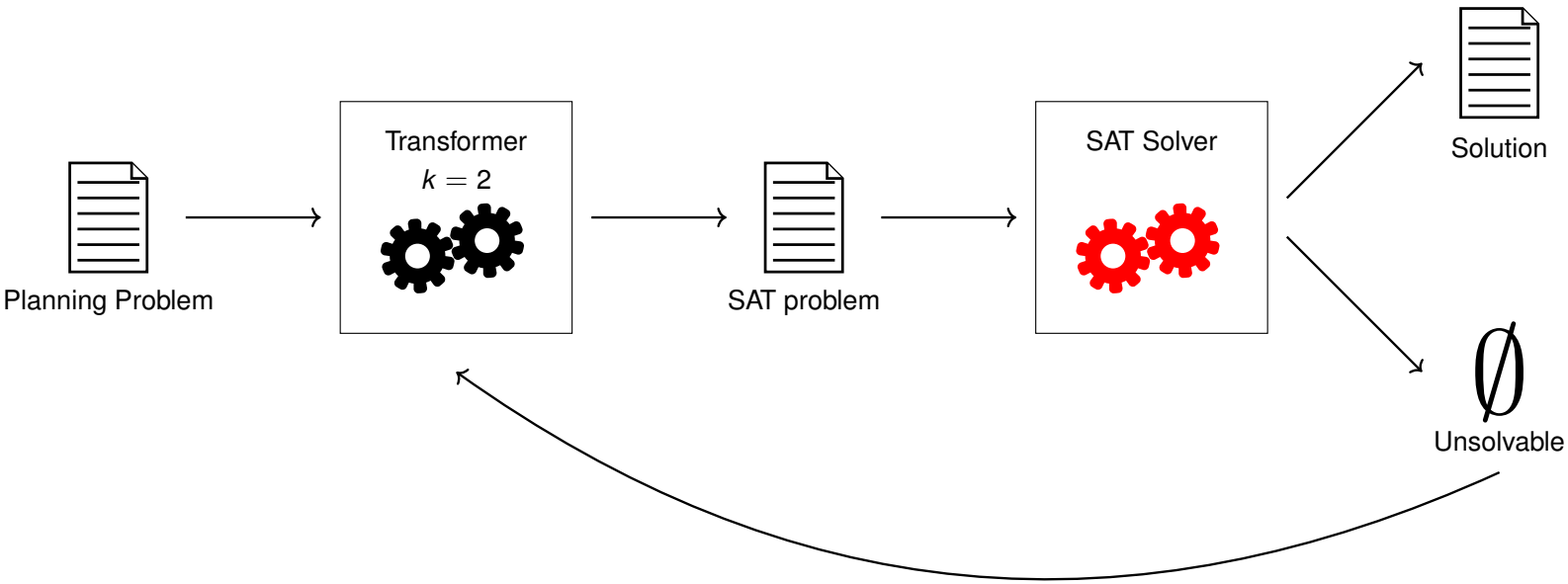

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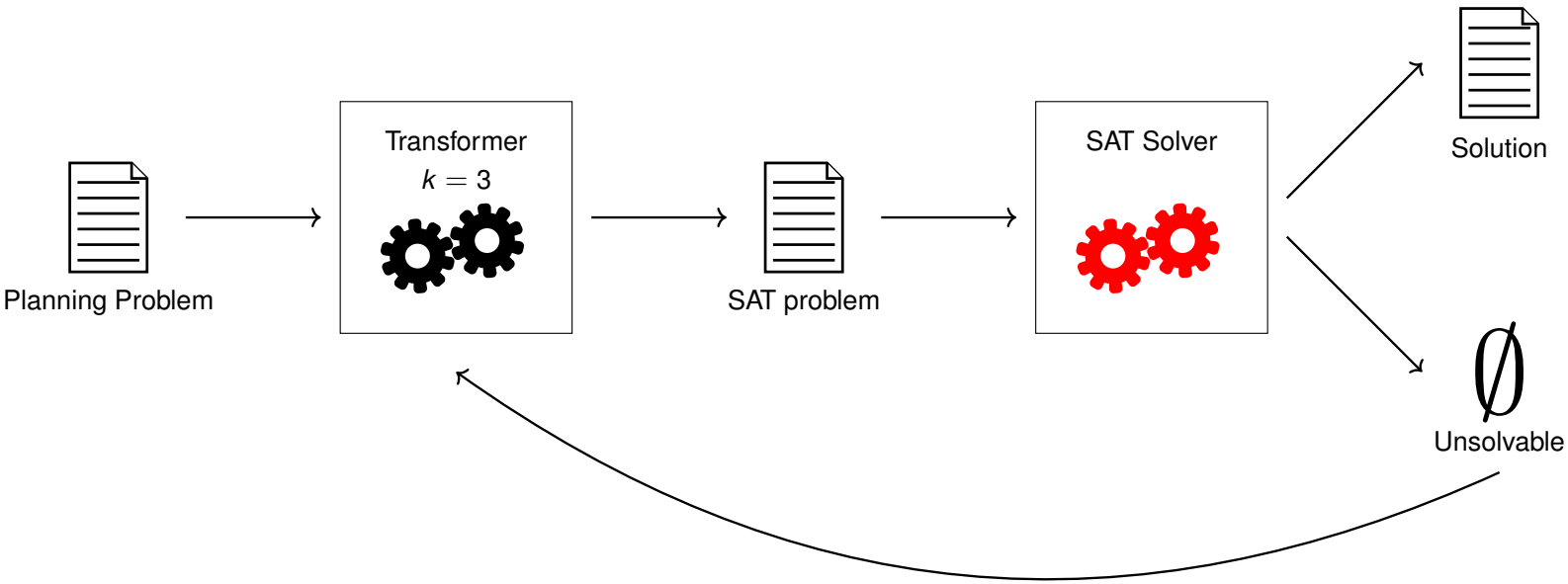

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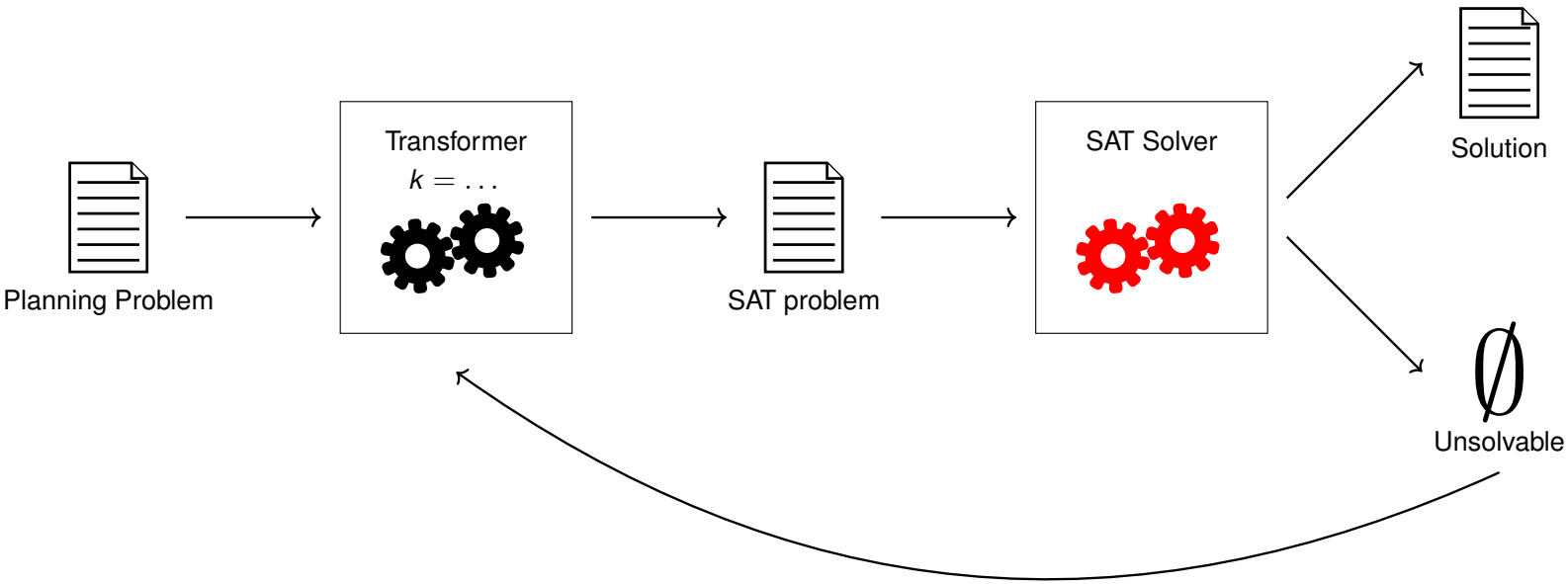

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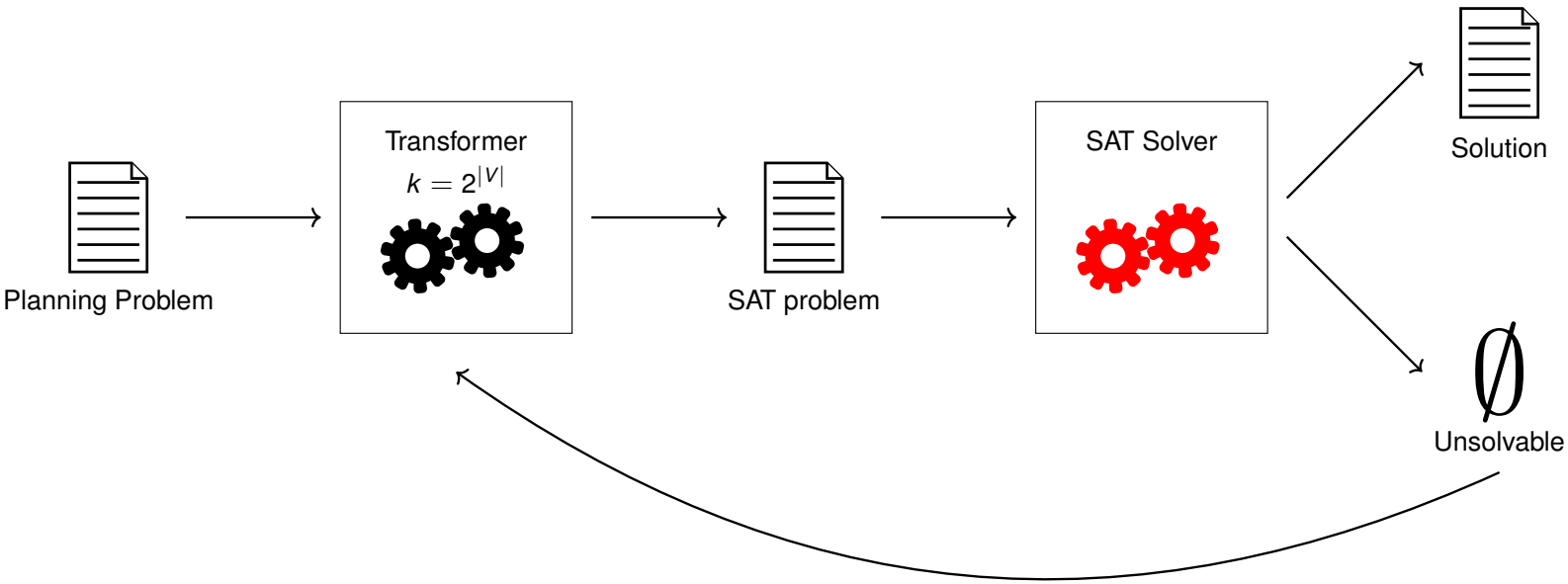
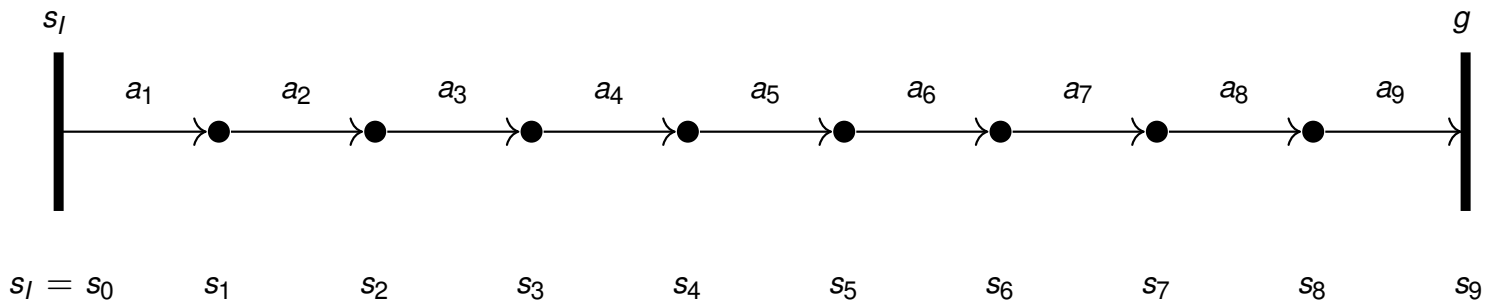


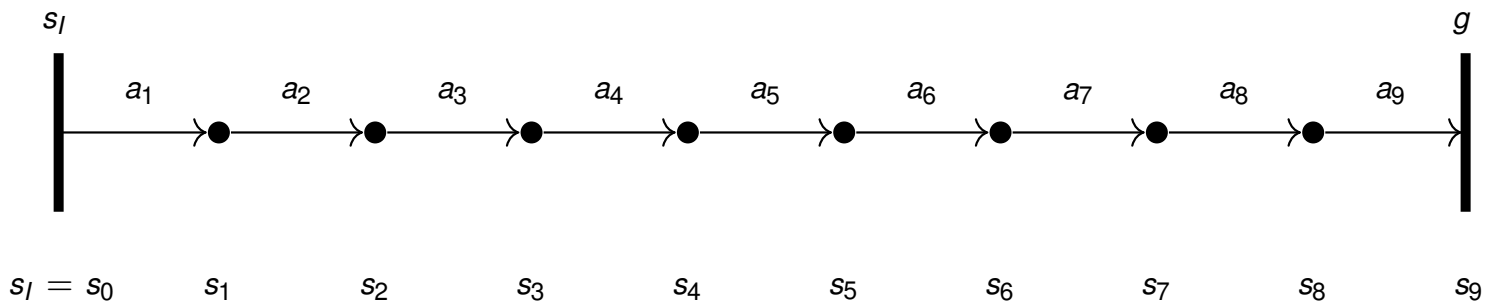
Classical Planning via SAT [Kautz&Selman'92]



A (classical) plan is just a sequence of state transitions.

- "Mechanics" is identical in all timesteps.
- Just model one timestep and copy'n'paste.
- Edge constraints!

Decision Variables



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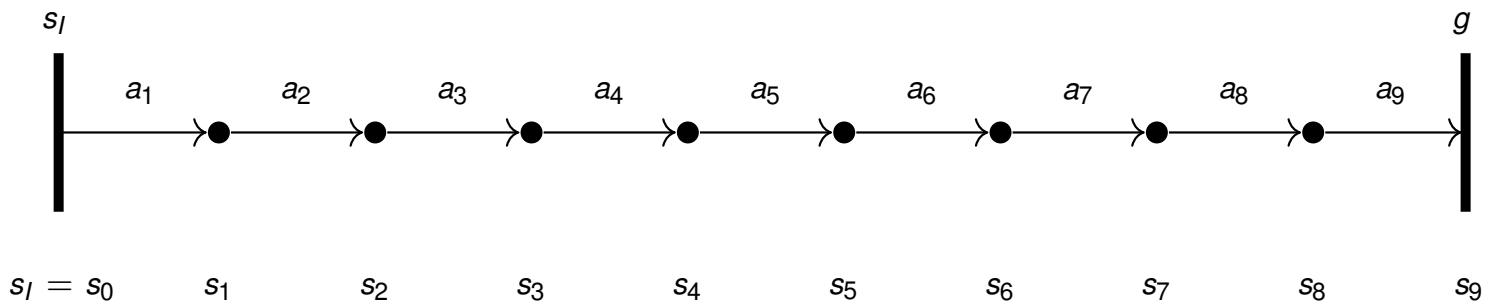

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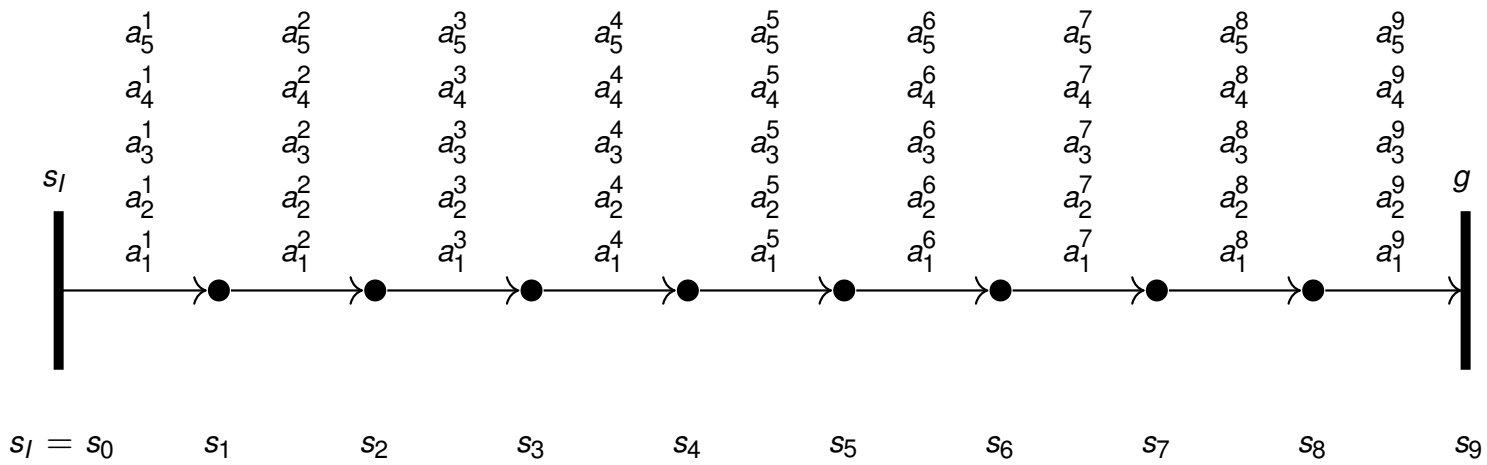

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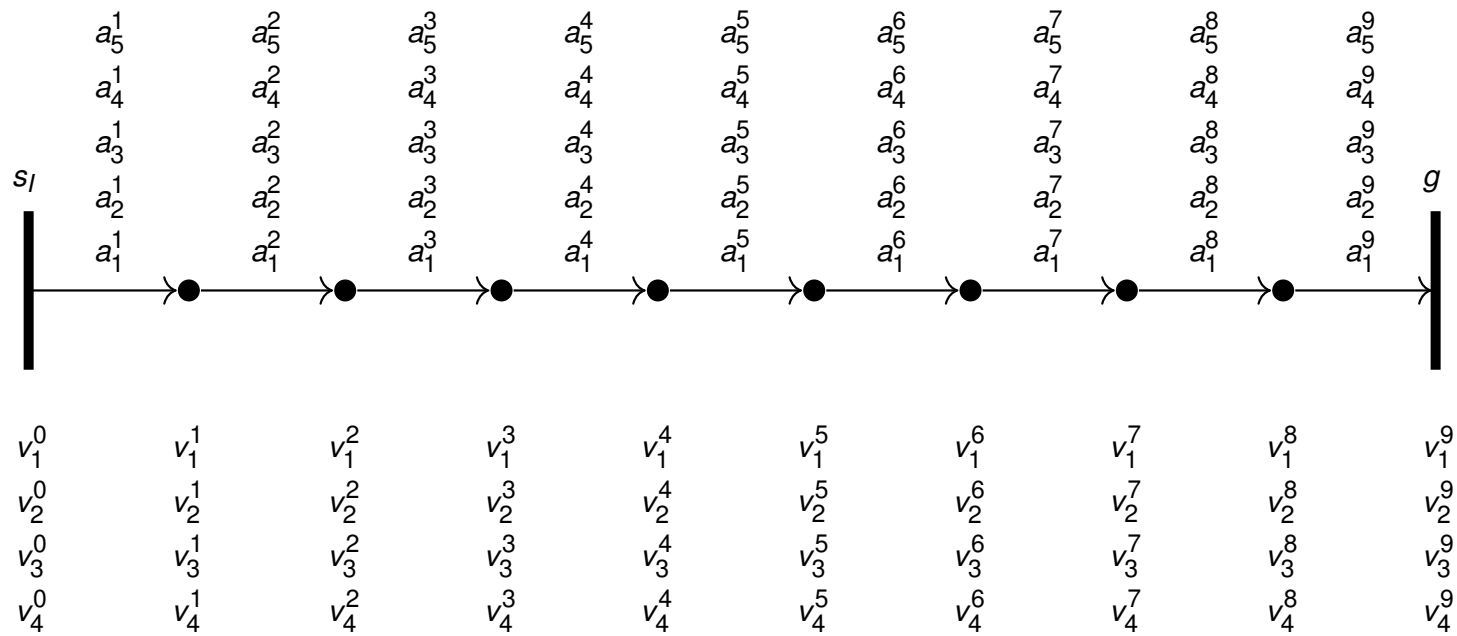

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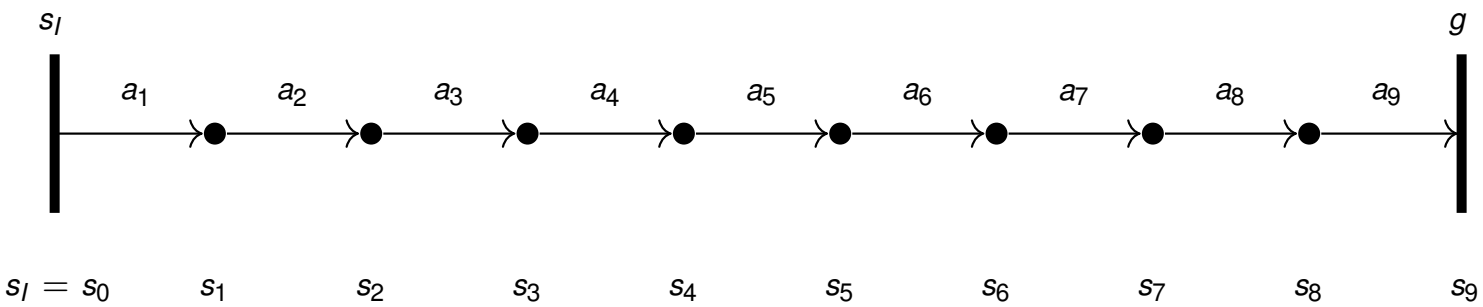
1. a_i^t – Action i is executed at time t.
2. v_i^t – State variable i is true at time t.

Overall Formula



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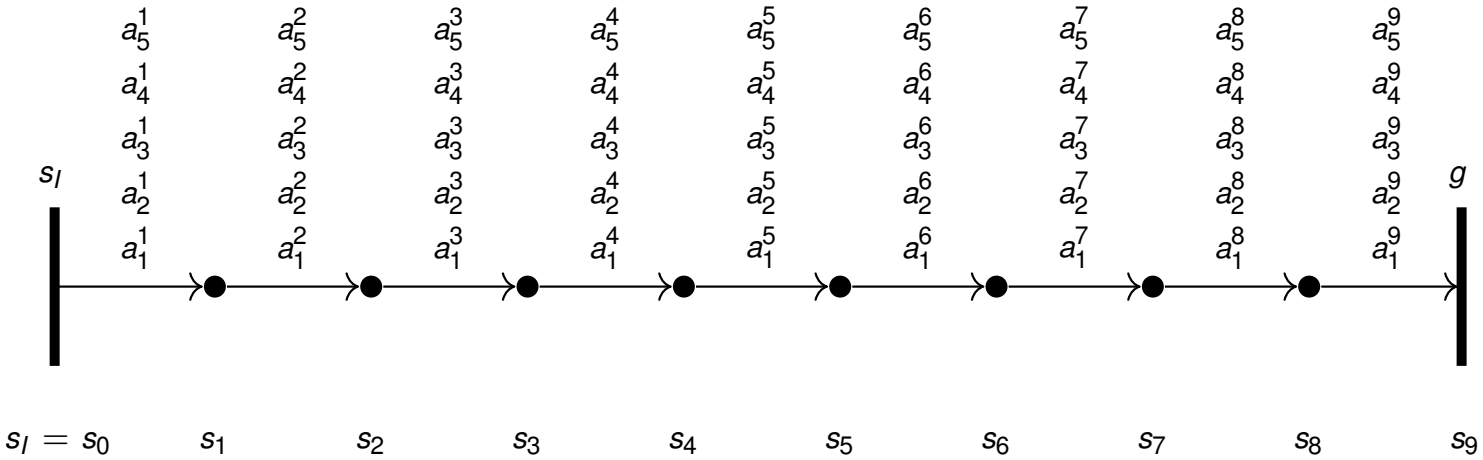

Constraints to check:

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$$\mathcal{F} = \bigwedge_{t=0}^{k-1} \tau(t)$$

here: $k = 9$

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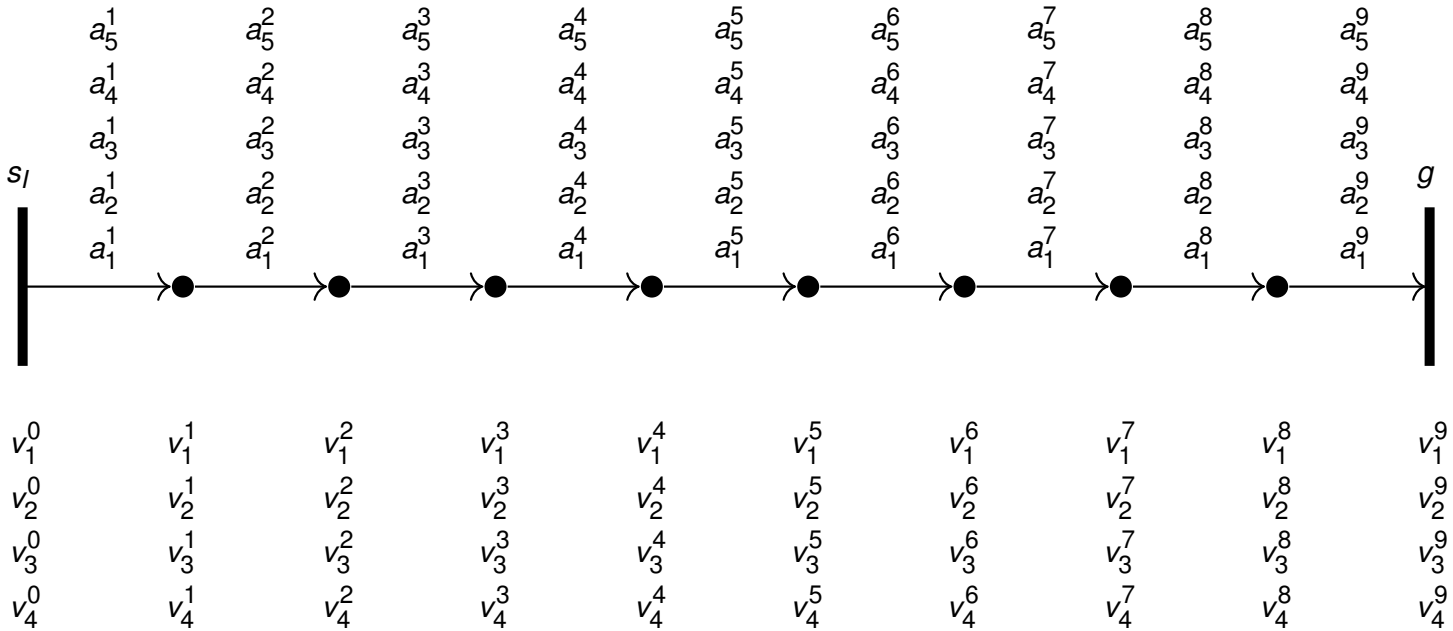
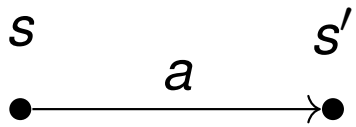


Constraints to check:

- Correctly applying actions at each time step (τ).
- s_I and g must be respected.

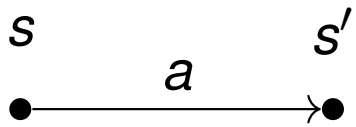
$$\mathcal{F} = \bigwedge_{t=0}^{k-1} \tau(t) \wedge \bigwedge_{v_i \in s_I} v_i^0 \wedge \bigwedge_{v_i \in V \setminus s_I} \neg v_i^0 \wedge \bigwedge_{v_i \in g} v_i^k \quad \text{here: } k = 9$$

Classical Planning via SAT



Constraints to check by $\tau(t)$:

Classical Planning via SAT



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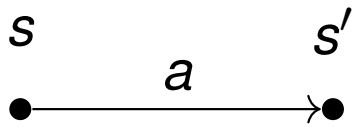

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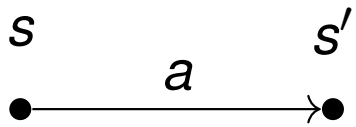

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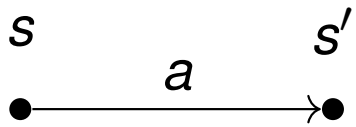

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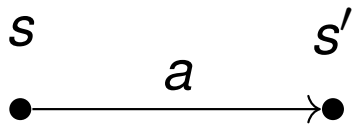

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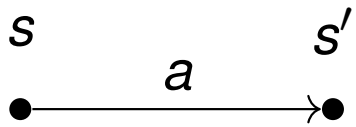

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- F_5 At least one action per timestep. Necessary? **No.**

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- Preconditions must hold:

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$$F_1 = \bigwedge_{a \in A} \left[a^{t+1} \rightarrow \bigwedge_{v \in pre(a)} v^t \right]$$

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- Effects must occur:

$$F_2 = \left[\bigwedge_{a \in A} \left(a^{t+1} \rightarrow \bigwedge_{v \in add(a)} v^{t+1} \right) \right] \wedge \left[\bigwedge_{a \in A} \left(a^{t+1} \rightarrow \bigwedge_{v \in del(a)} \neg v^{t+1} \right) \right]$$

Classical Planning via SAT

- Variables not affected by the executed action must stay the same.

- Only one action at a time:

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  → Frame Problem!

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$$F_3 = \bigwedge_{v \in V} \left[(\neg v^t \wedge v^{t+1}) \rightarrow \bigvee_{a \in A \text{ with } v \in add(a)} a^{t+1} \right] \wedge$$

$$\bigwedge_{v \in V} \left[(v^t \wedge \neg v^{t+1}) \rightarrow \bigvee_{a \in A \text{ with } v \in del(a)} a^{t+1} \right]$$

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$$\bigwedge_{v \in V} \left[(v^t \wedge \neg v^{t+1}) \rightarrow \bigvee_{a \in A \text{ with } v \in del(a)} a^{t+1} \right]$$

- Only one action at a time:

$$F_4 = \text{at-most-one}(\{a^t \mid a \in A\})$$

At-most-one

Given a set of decision variables $X = \{x_1, \ldots, x_{|X|}\}$. Find a set of clauses that, if satisfied, will ensure that at most one $x \in X$ is true.

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Naive encoding:

$$\bigwedge_{x_1 \in X} \bigwedge_{x_2 \in X \setminus \{x_1\}} \underbrace{\neg x_1 \vee \neg x_2}_{(x_1 \Rightarrow \neg x_2) \wedge (x_2 \Rightarrow \neg x_1)}$$

At-most-one

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Sequential encoding:

$$\bigwedge_{i=1}^{|X|-1} \underbrace{\neg x_i \vee f_i}_{x_i \Rightarrow f_i}$$

$$\bigwedge_{i=2}^{|X|-1} \underbrace{\neg f_{i-1} \vee f_i}_{f_{i-1} \Rightarrow f_i}$$

$$\bigwedge_{i=1}^{|X|} \underbrace{\neg x_i \vee \neg f_{i-1}}_{(x_i \Rightarrow \neg f_{i-1}) \wedge (f_{i-1} \Rightarrow \neg x_i)}$$

At-most-one

Maybe this is a bit much ...

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n_i – bit i (0-index) of a $\lceil \log(|X|) \rceil$-digit binary number if one

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Binary encoding:

$$\neg x_i \vee n_j \qquad \text{if } \frac{i}{2^j} \mod 2 = 1$$

$$\neg x_i \vee \neg n_j \qquad \text{if } \frac{i}{2^j} \mod 2 = 0$$

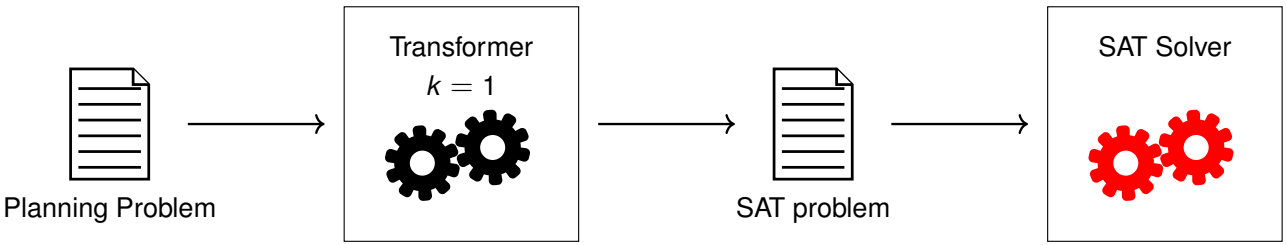
Different AMO Implementations[1]

encoding	#clauses	#new variables
binomial	n^2	0
binary	$n \log n$	$\log n$
sequential	$3n$	n
commander	$\frac{7}{2}n$	$\frac{n}{2}$
product	$2(n + n^{\frac{1}{m+1}})$	$2n^{\frac{1}{2}}$

where n is the number of atoms, i.e., $|X|$

[1] Frisch and Giannaros; SAT Encodings of the At-Most-k Constraint – Some Old, Some New, Some Fast, Some Slow; 2010

Bound Iteration



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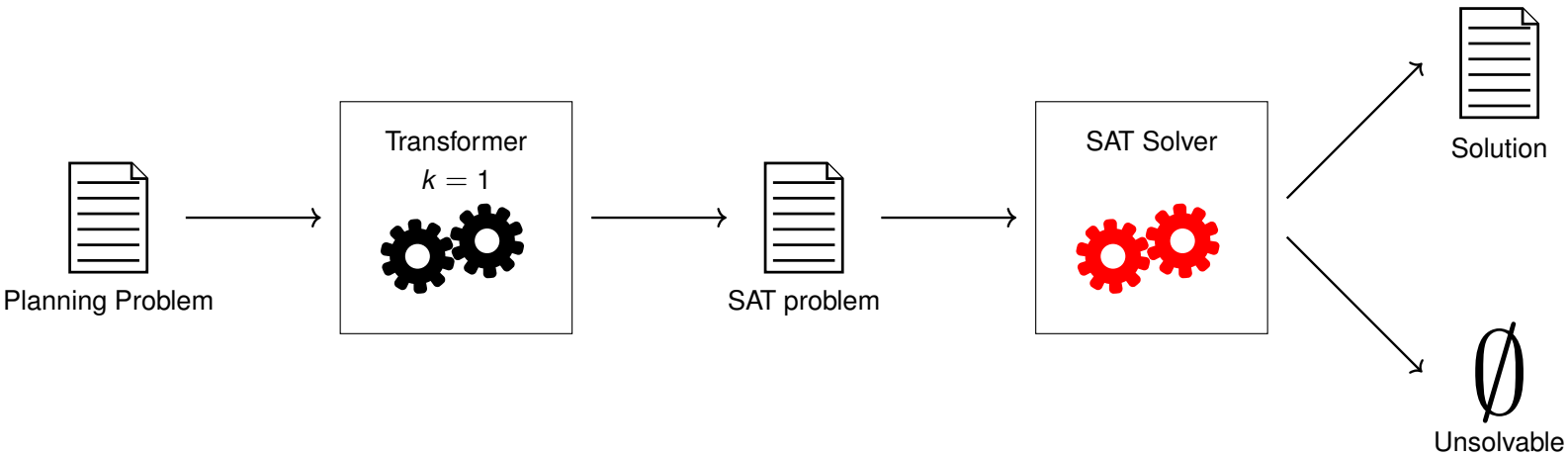

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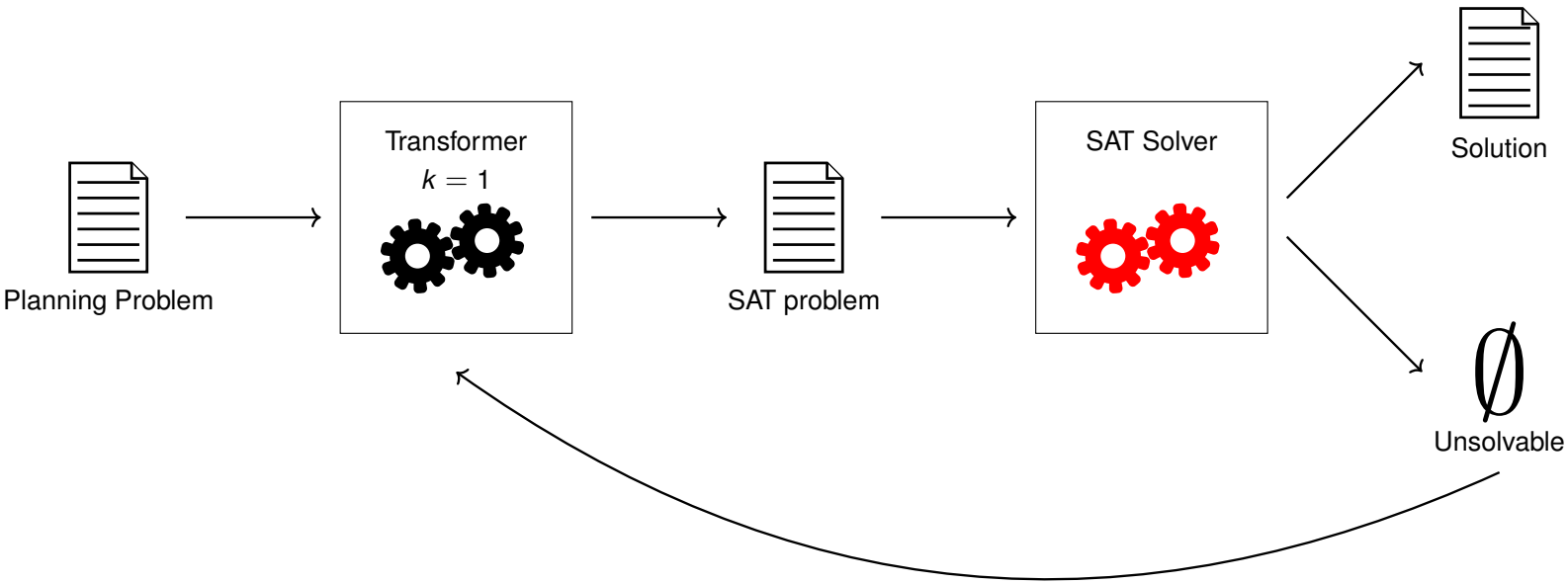

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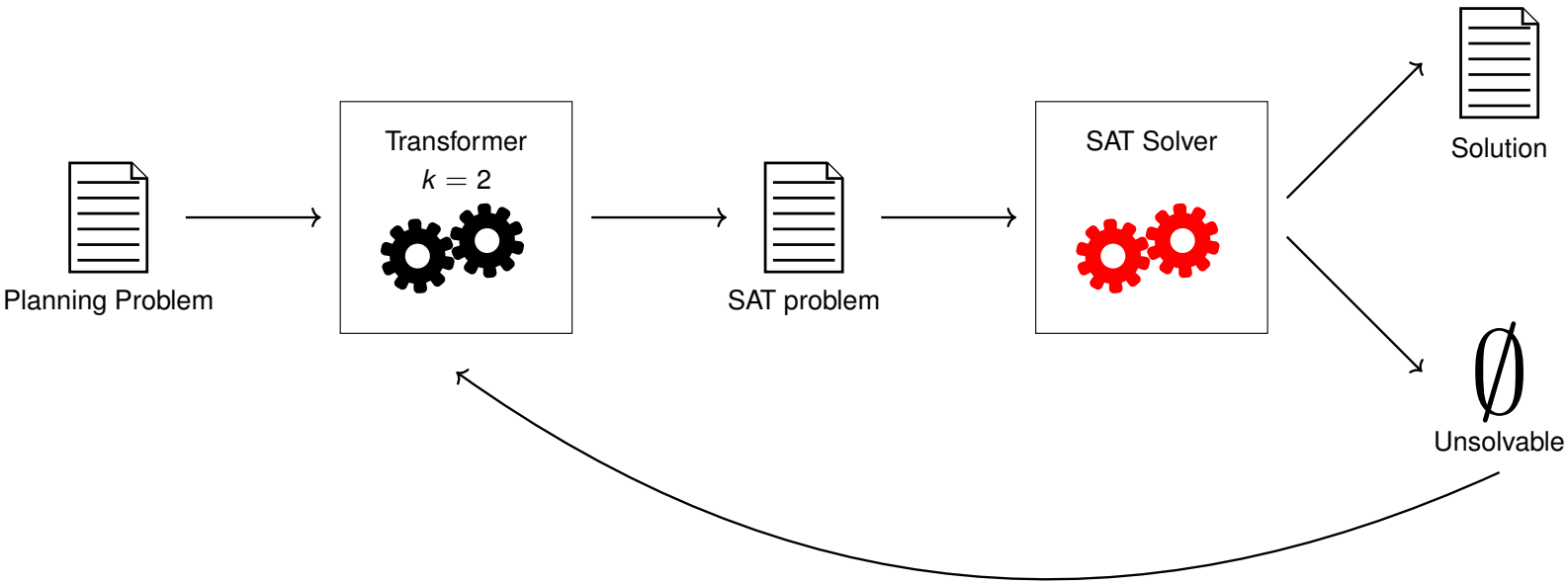

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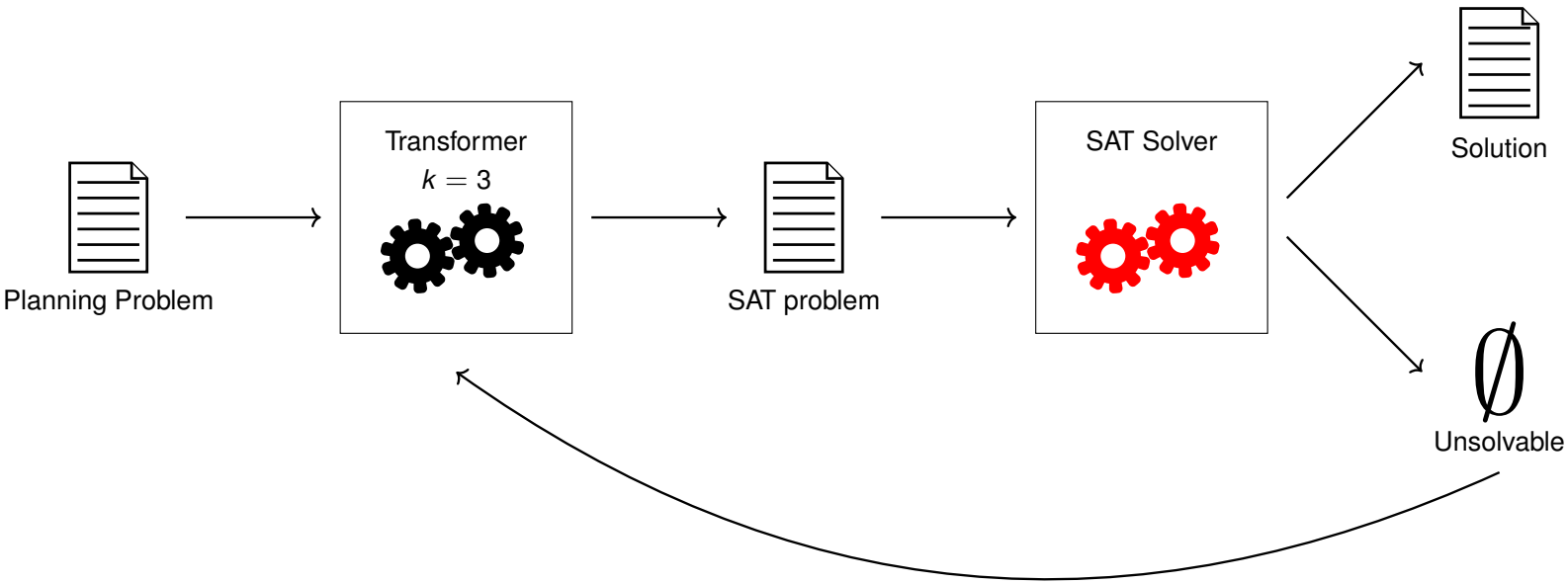

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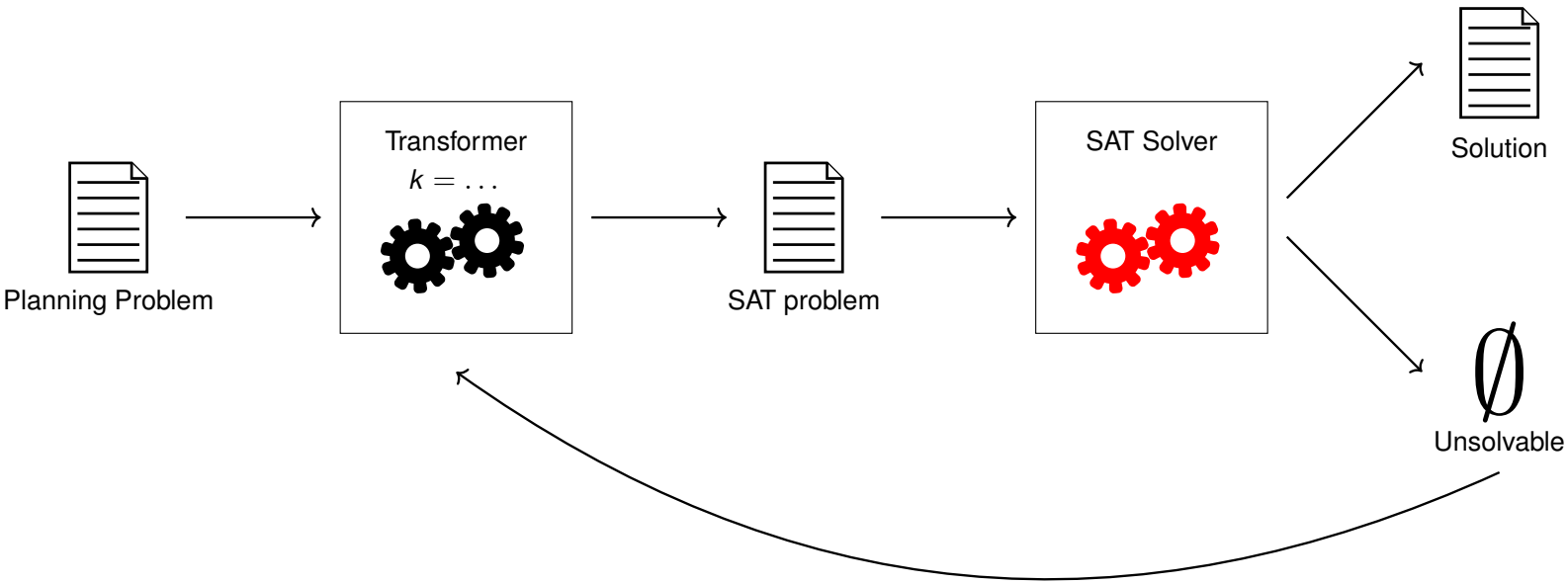

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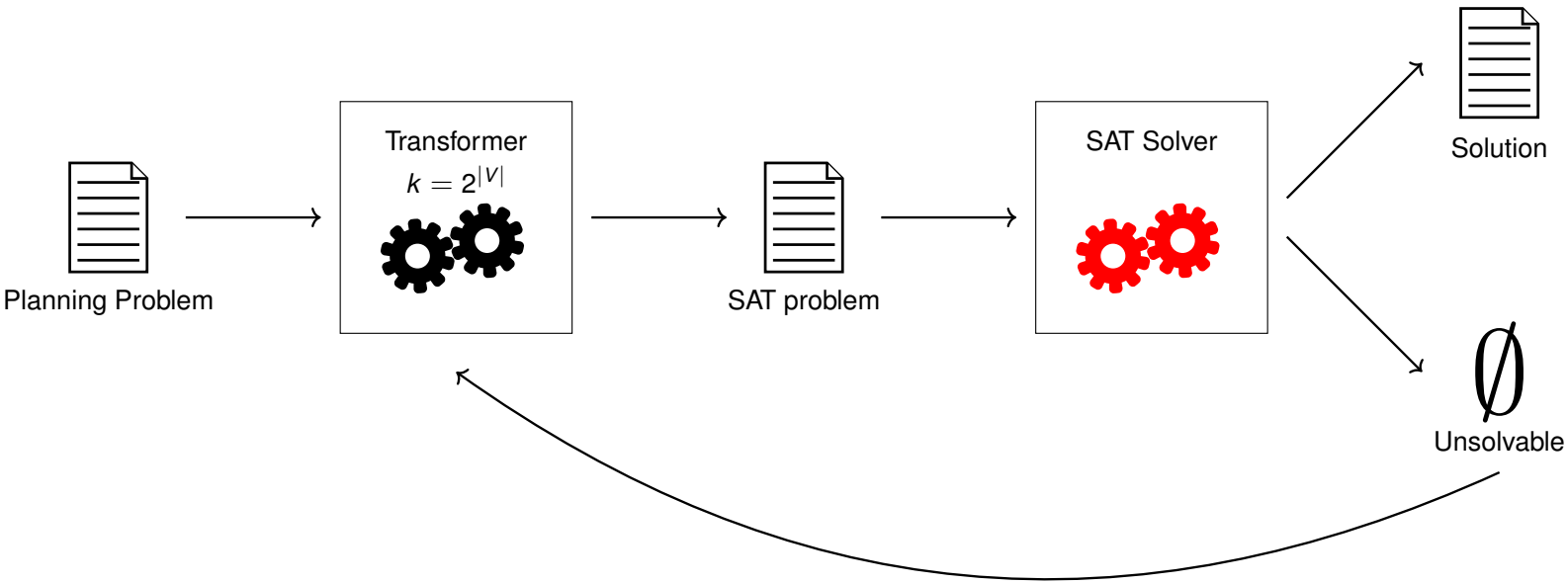

Classical Planning via SAT

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What are Invariants?

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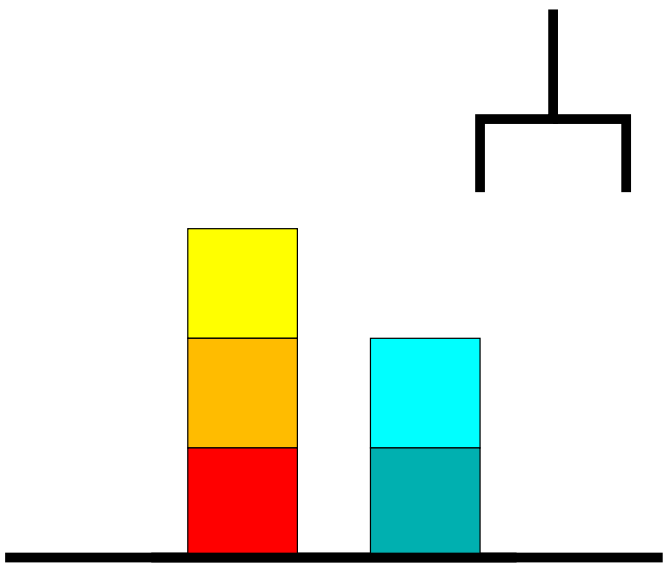
Definition (Invariant)

An invariant $\mathcal{I}$ is a formula over the state variables such that for all states s reachable from s_I it holds $s \models \mathcal{I}$.

What are Invariants?



Predicates:

- $on(x, y)$ – x lies directly on y.
- $free(x)$ – x has no block above it.

Actions:

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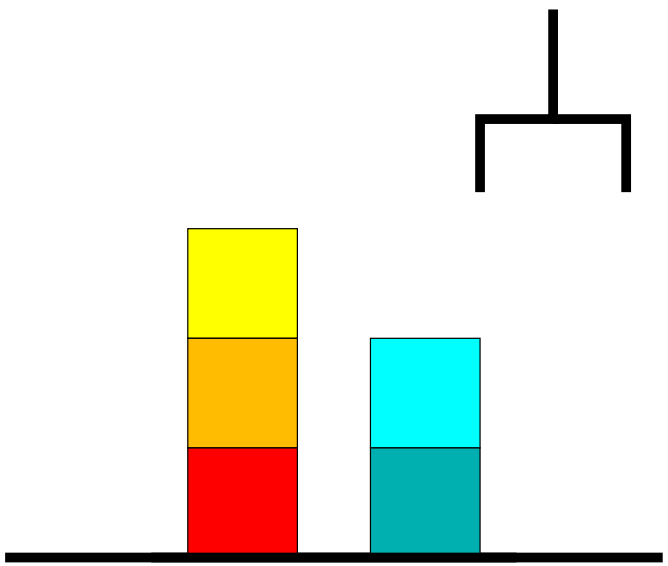

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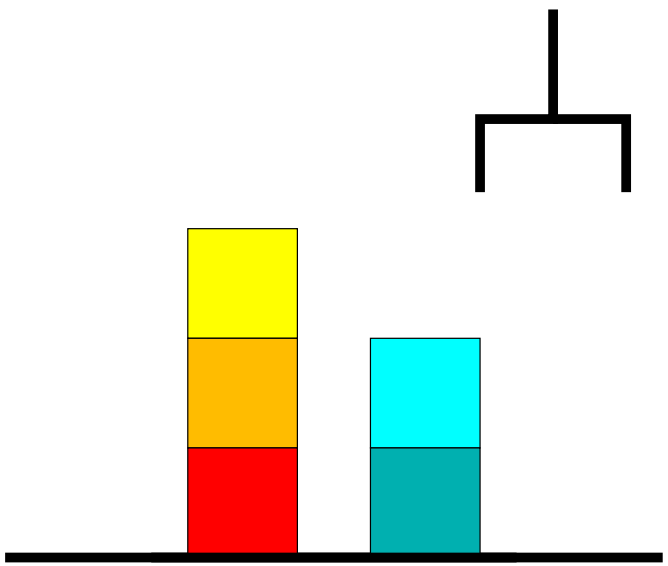

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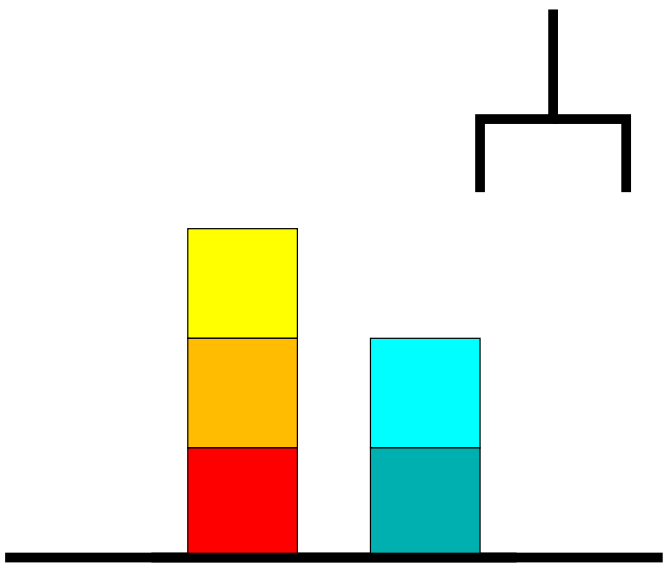

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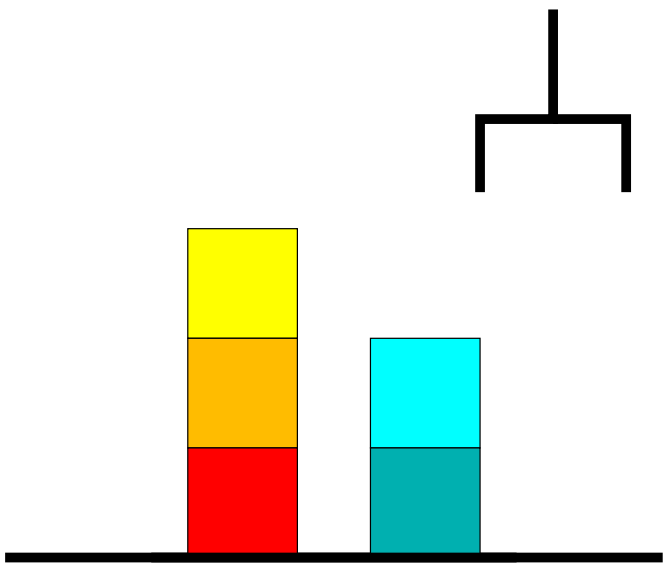

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- Compute an approximation of all invariants of a fixed form.
- Restrict to binary-or invariants:

$$\ell_1 \vee \ell_2$$

Computing Invariants [Rintanen'98]

Note: Here we consider some action $a = (pre, add, del)$ and denote with $eff = add(a) \cup \{\neg v \mid v \in del(a)\}$ its effects (as a literal set).

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$U_{\langle pre,eff\rangle}(\mathcal{I})$ gives all properties (positive or negative state variables) that hold after the execution of an action $a = \langle pre, eff\rangle$

$$U_{\langle pre,eff\rangle}(\mathcal{I}) = (\{\ell \in V \cup \neg V \mid \mathcal{I} \cup pre \models \ell\} \setminus \overbrace{\{\neg\ell \mid \ell \in eff\}}^{\equiv(\{\neg v \mid v \in add\} \cup del)}) \cup eff$$

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$F_{\langle pre,eff\rangle}(\mathcal{I})$ is a *filter* for invariants, returning those that hold after the execution of an action $a = \langle pre, eff\rangle$

$$F_{\langle pre,eff\rangle}(\mathcal{I}) = \begin{cases} \mathcal{I} & \text{if } \mathcal{I} \cup pre \models \bot \text{ and otherwise:} \\ \{\ell_1 \vee \ell_2 \in \mathcal{I} \mid \left(\neg\ell_1 \notin eff \text{ or } \ell_2 \in U_{\langle pre,eff\rangle}(\mathcal{I})\right) \text{ and} & \\ \qquad \left(\neg\ell_2 \notin eff \text{ or } \ell_1 \in U_{\langle pre,eff\rangle}(\mathcal{I})\right)\} & \end{cases}$$

Computing Invariants [Rintanen'98], cont'd

Call $R_A(\mathcal{I}) := F_{a_1}(F_{a_2}(\cdots F_{a_n}(\mathcal{I})\cdots))$ with initial invariant

$I_{init} = \{v \vee \ell \mid v \in s_I, \ell \in V \cup \neg V\} \cup \{\neg v \vee \ell \mid v \notin s_I, \ell \in V \cup \neg V\}$

and arbitrary linearization of action set A, $a_1, \ldots, a_n$,

until $\mathcal{I}$ does not change anymore.

R stands for "*reduce* invariant set".

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What to do with an invariant $\ell_1 \vee \ell_2$?

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Add it to every timestep t as $\ell_1^t \vee \ell_2^t$.

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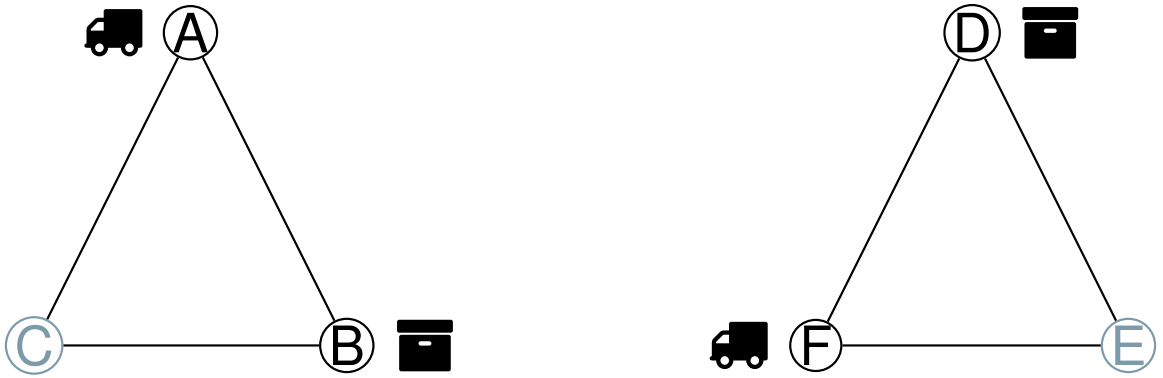

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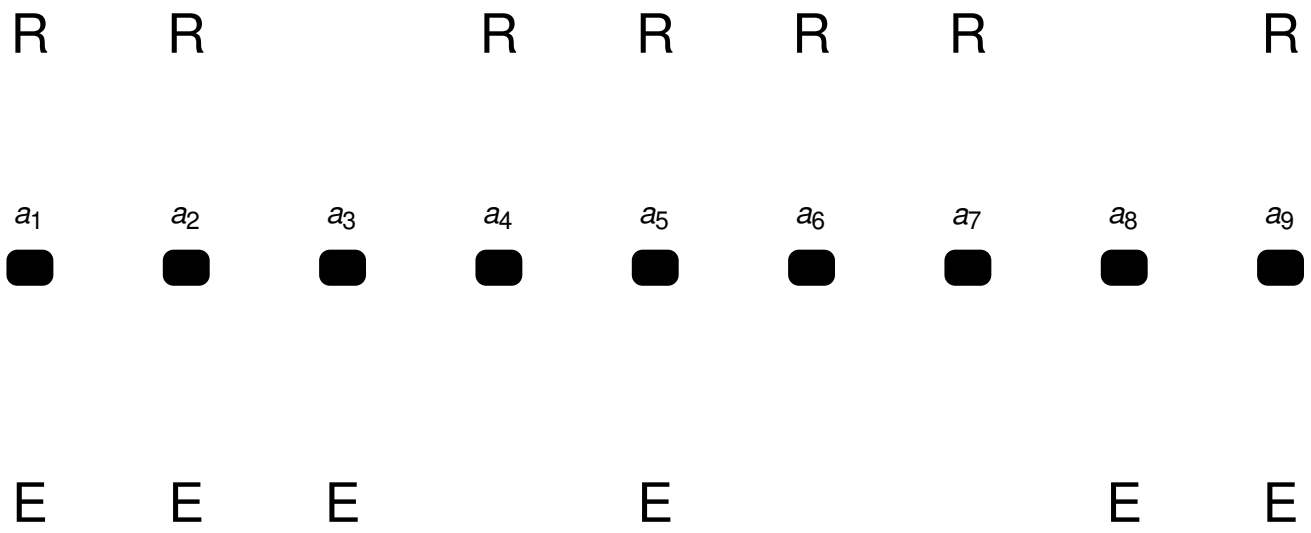
E	E	E		E			E	E
R	R		R	R	R	R		R

- E_v – subsequence of S with $v \in del(a)$ (**E**rasing)
- R_v – subsequence of S with $v \in pre(a)$ (**R**equiring)

Chains



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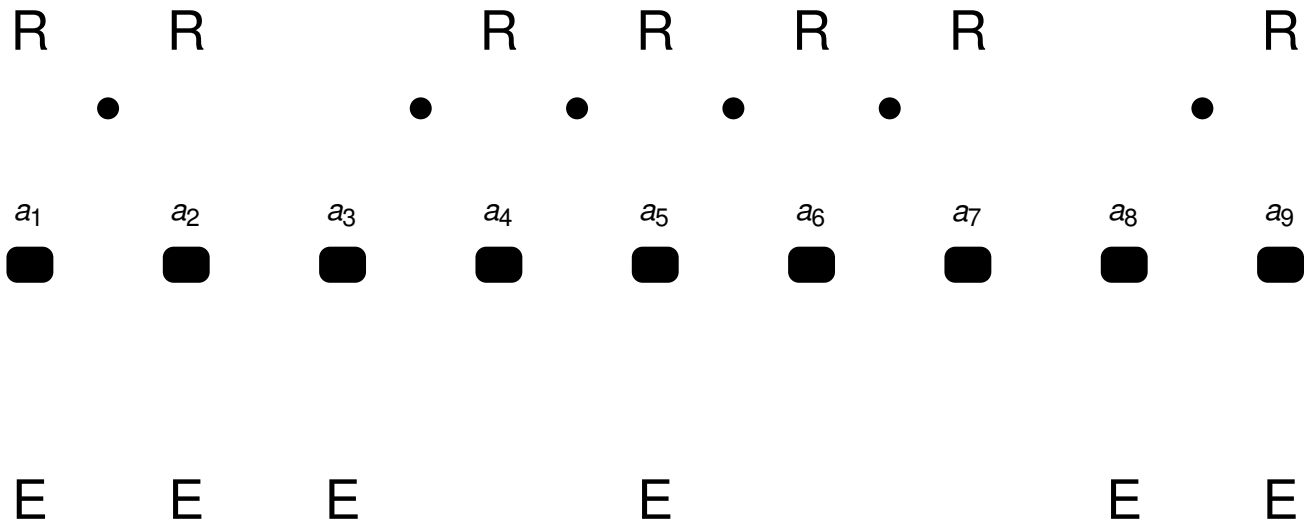

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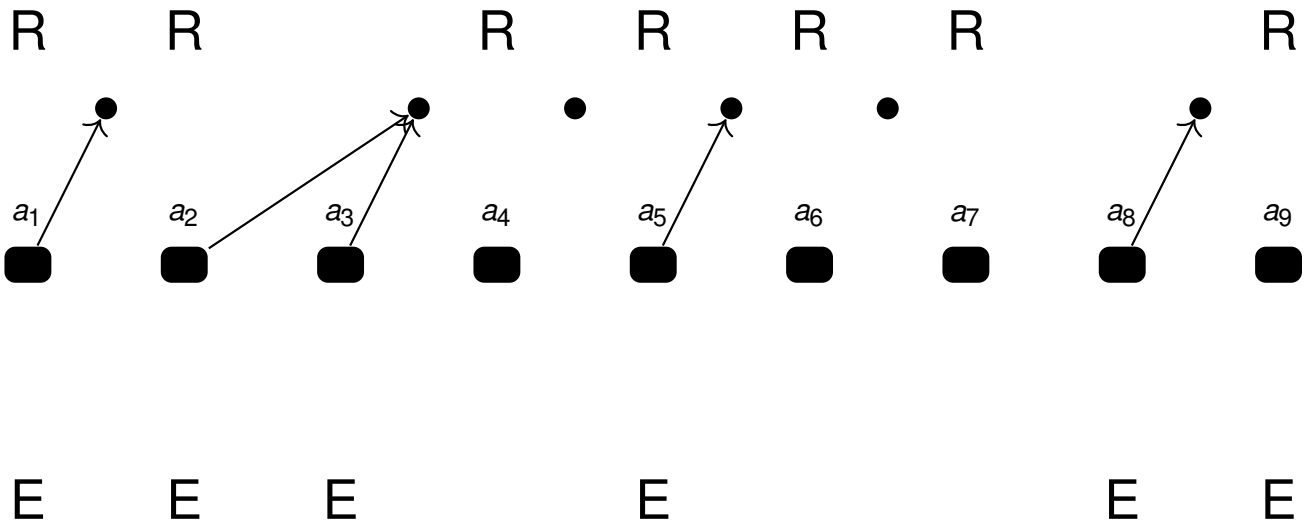

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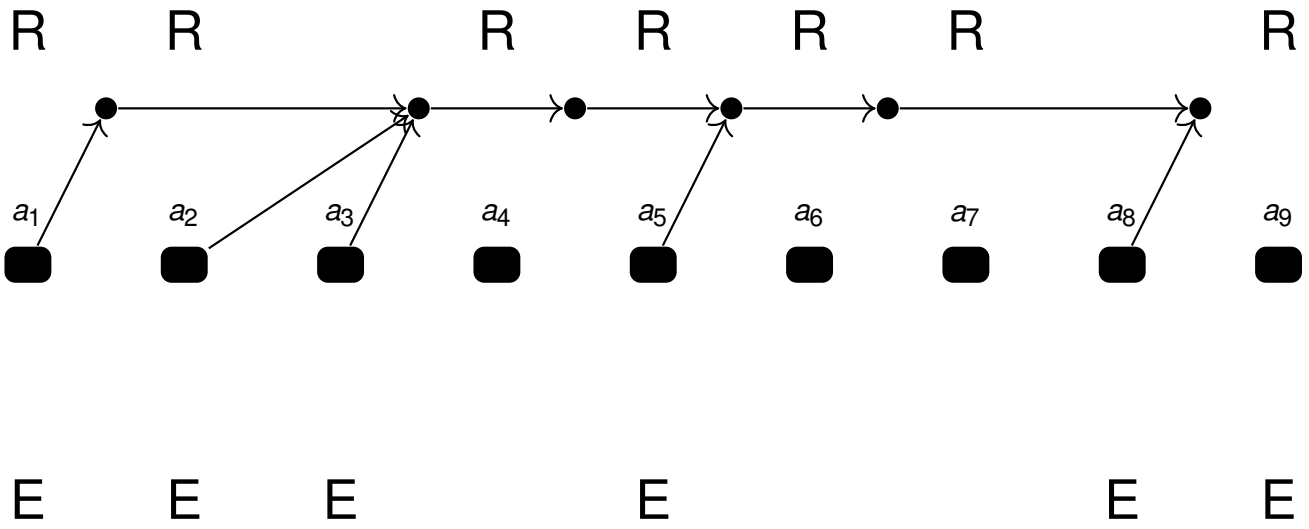

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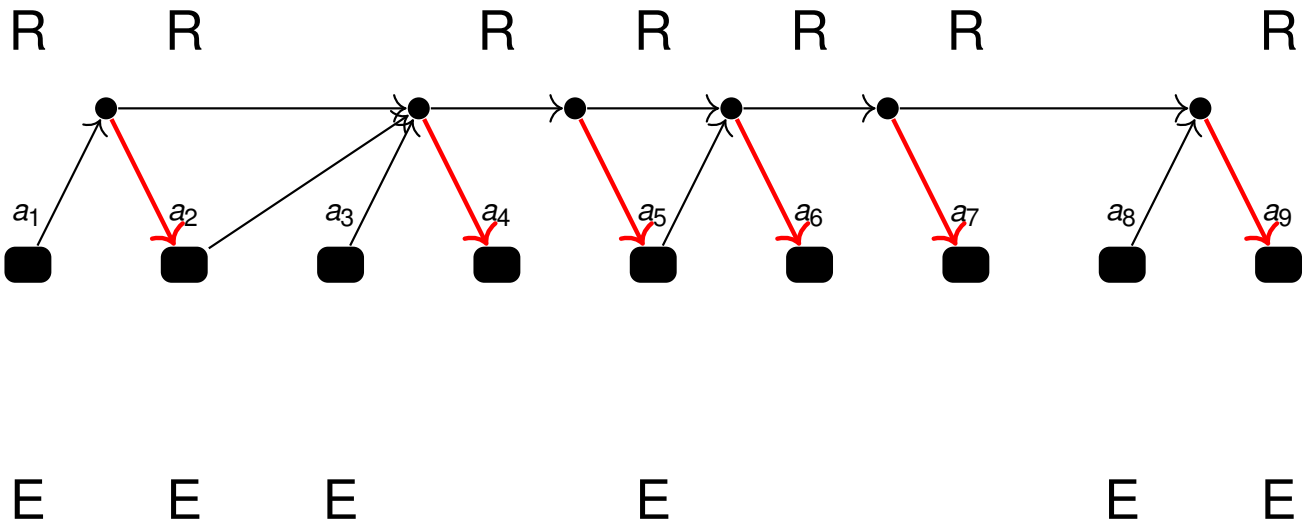

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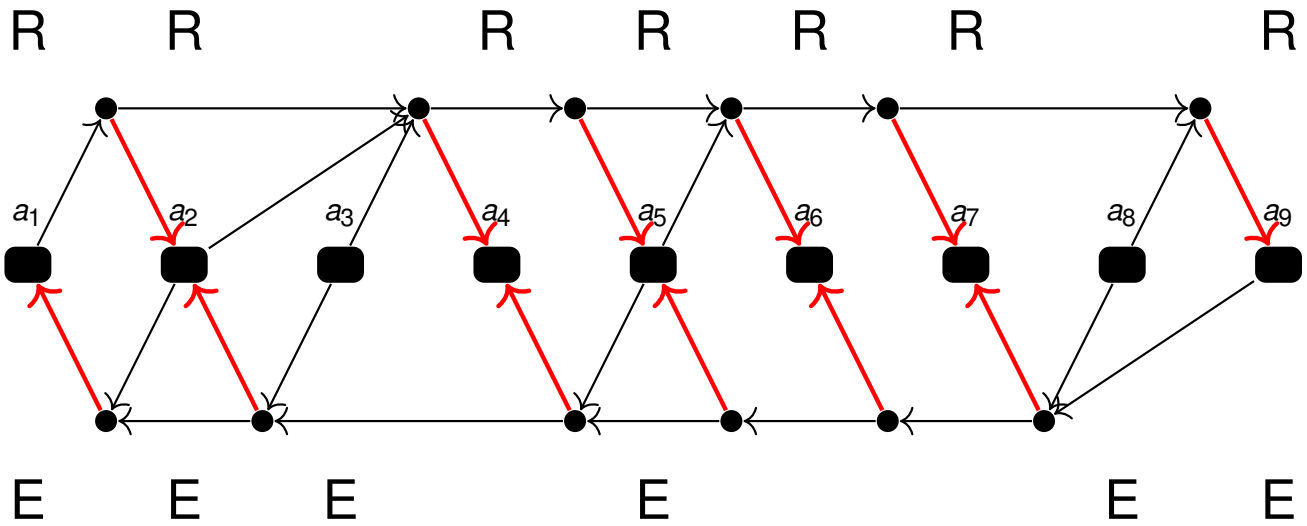

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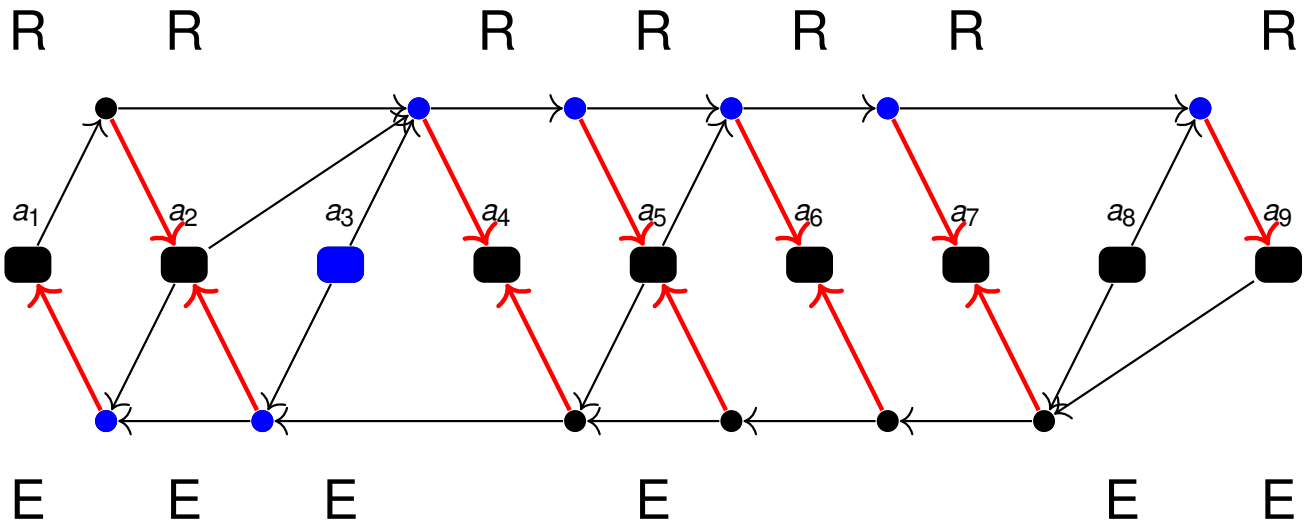

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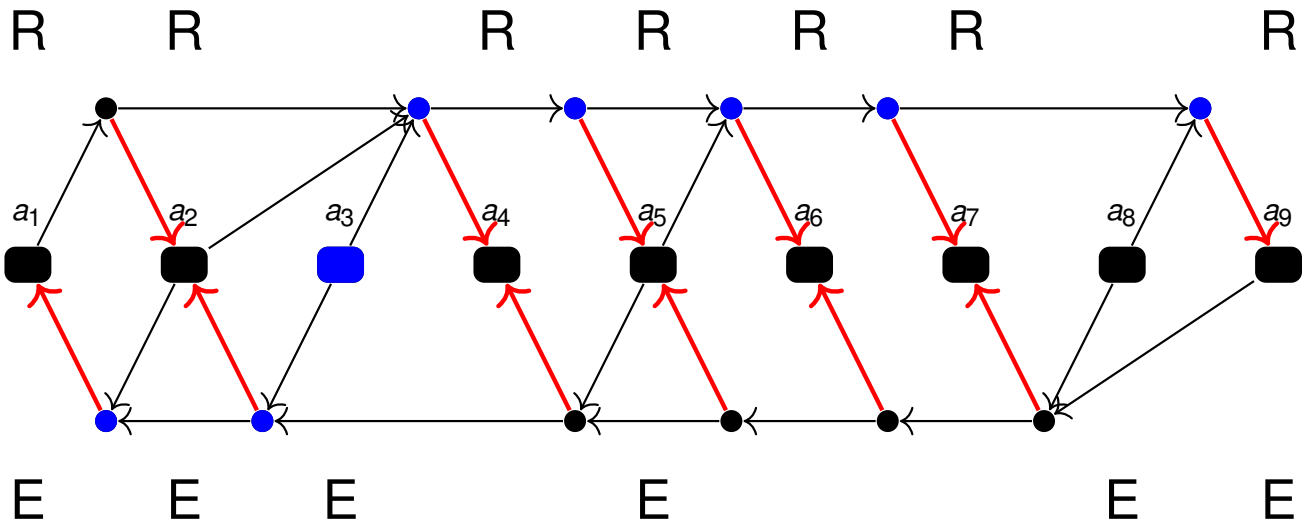

$$chain(S, E, R) =$$

$$\bigwedge \{a^i \rightarrow \mathtt{f}^j \mid i < j, a^i \in E, a^j \in R, \{a_{i+1}, \ldots, a_{j-1}\} \cap R = \emptyset\} \cup$$
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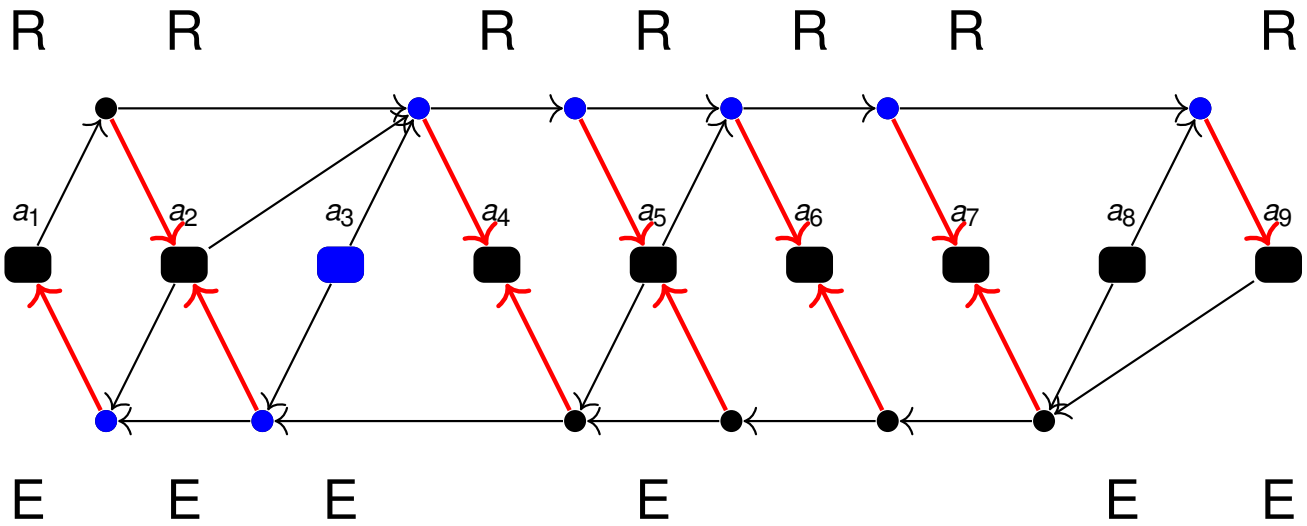

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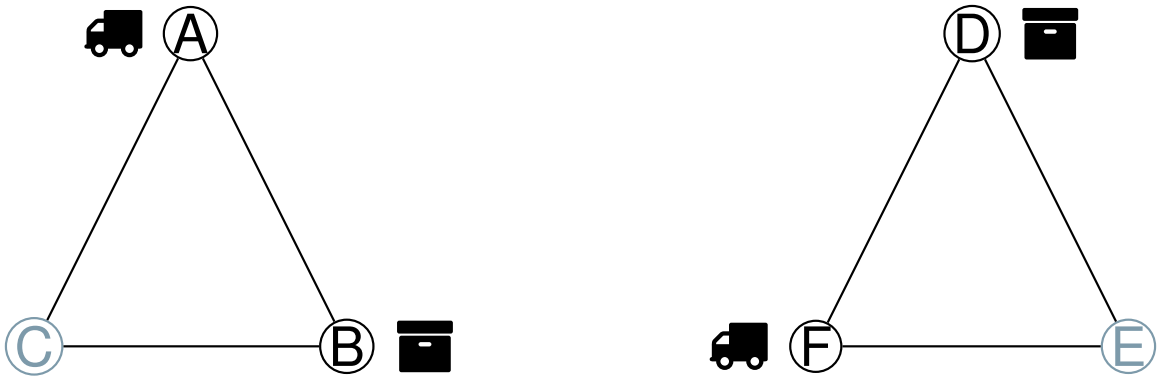
Two chains for every $v \in V$ with fresh decision variables $\mathtt{f}^i$.

Parallel Plans are (Still) Bad!

(Re-)Consider the following (single) planning problem:



drive(A, B) load(B) drive(B, C) unload(C)
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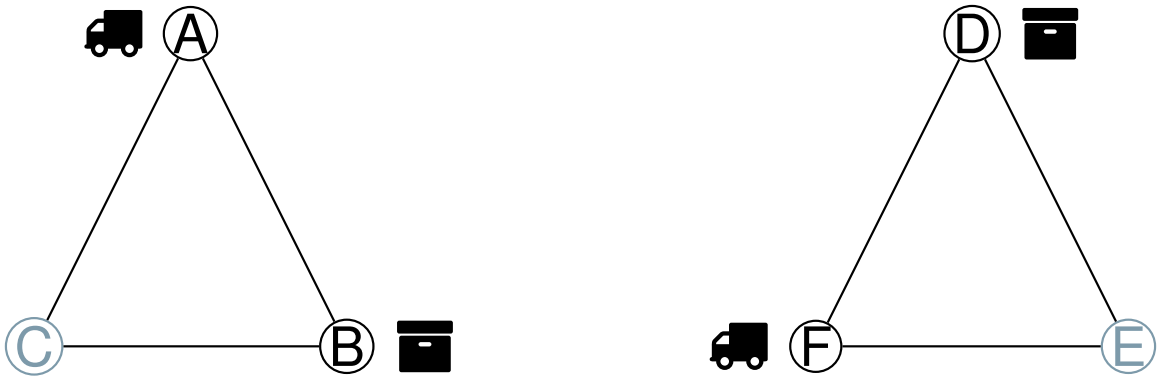

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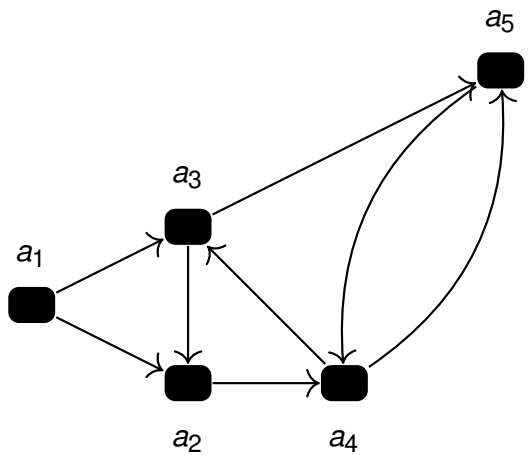

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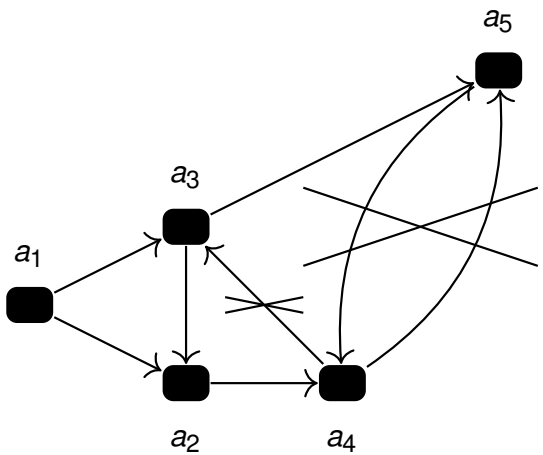

∃-step [Rintanen,Heljanko,Niemelä'06]

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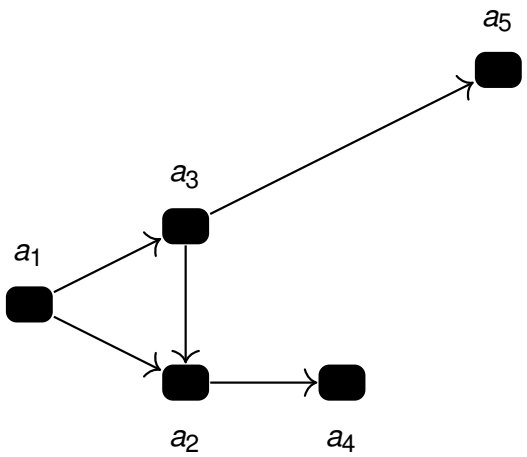

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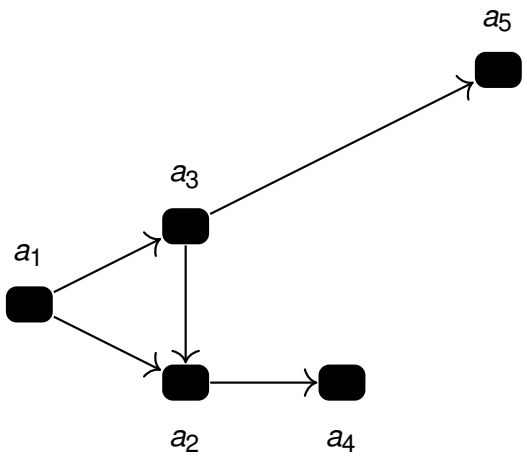

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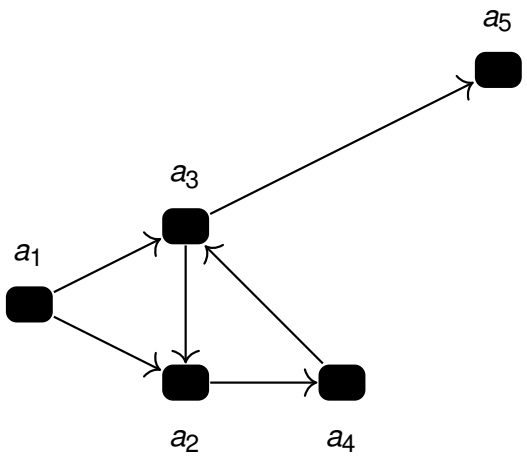

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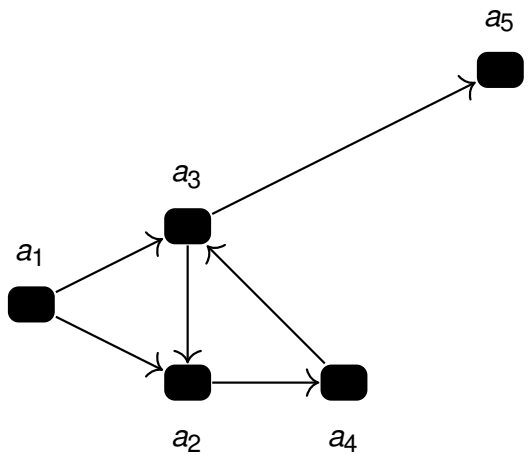

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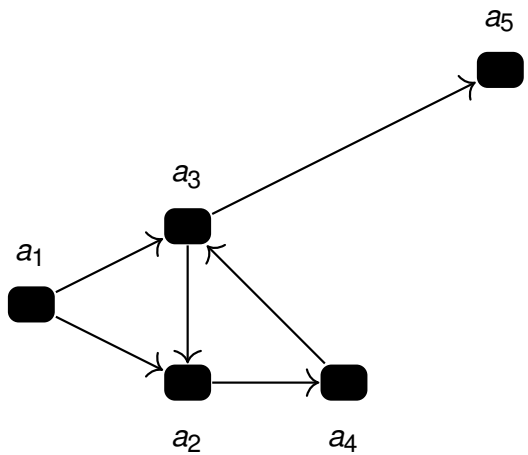
- If executed in parallel, we will always execute actions in **this** order.



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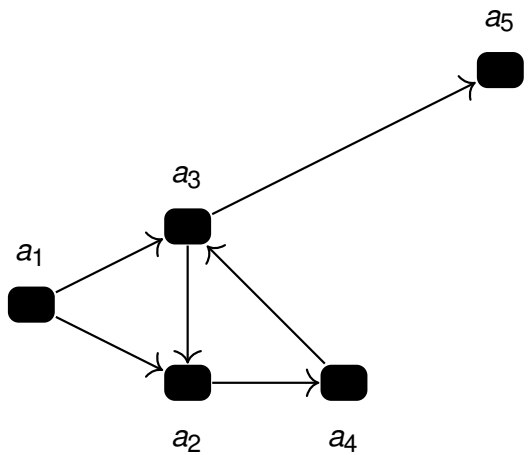

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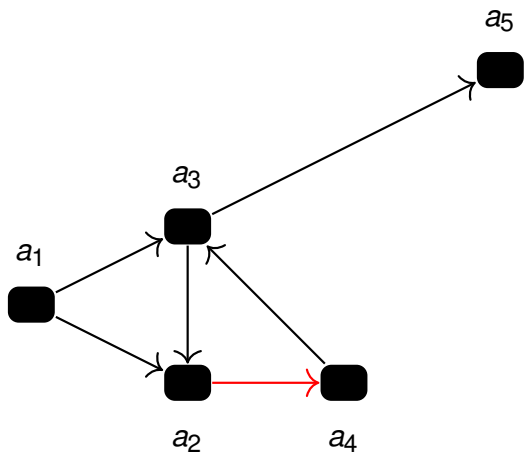

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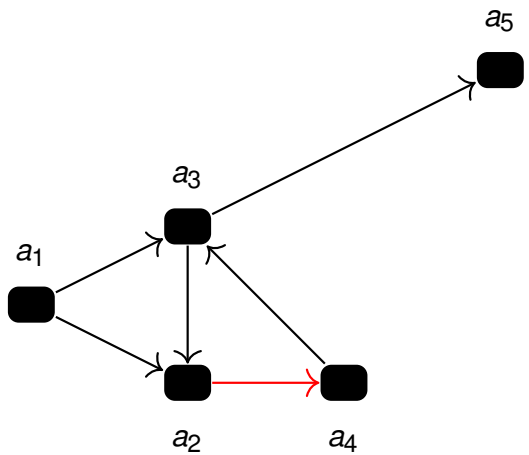

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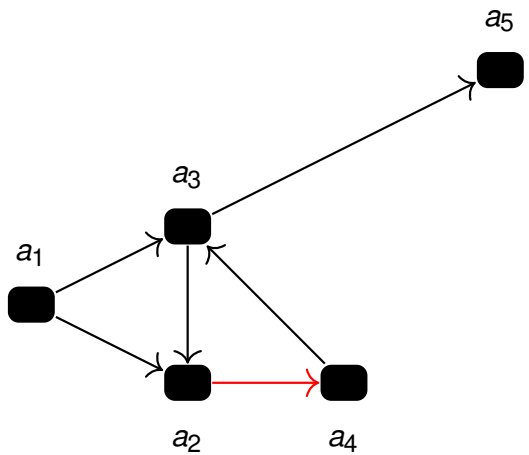

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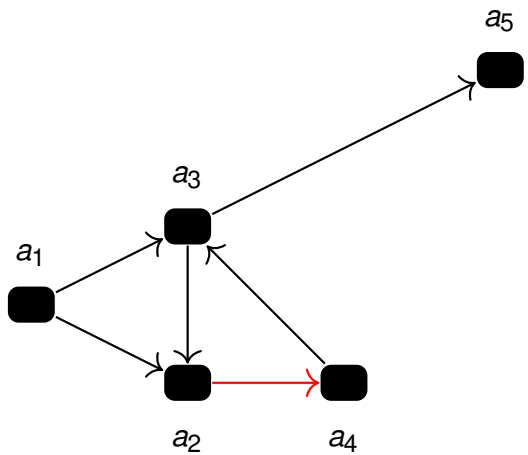
  6. Enforced via *chaines*.



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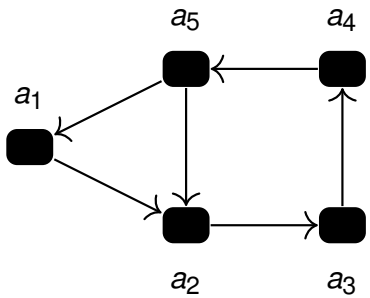
Chains

We are given an SCC and an ordering of its vertices.



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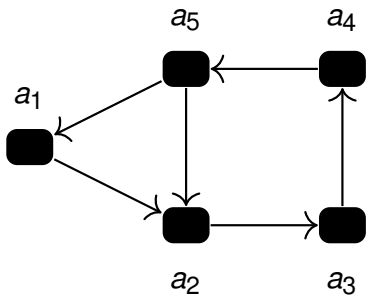

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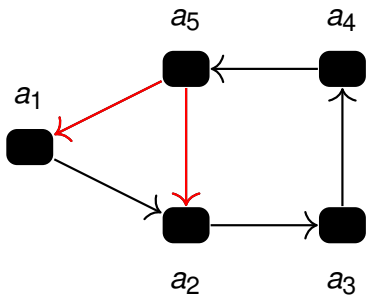

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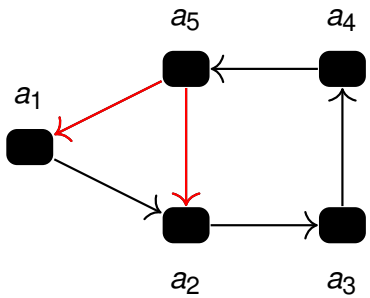

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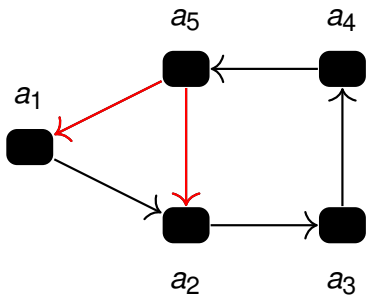

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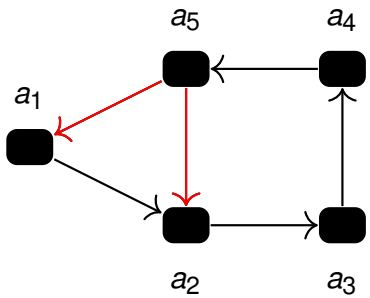

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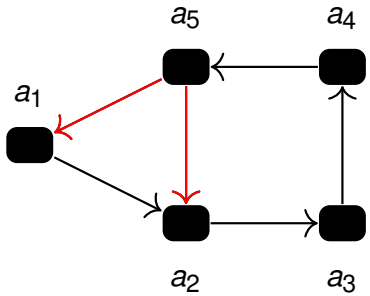

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  - E_v – subsequence of π with $v \in del(a)$ (**E**rasing)
  - R_v – subsequence of π with $v \in pre(a)$ (**R**equiring)
- Add $chain(\pi, E_v, R_v)$ – i.e. whenever an action erases v, we forbid any requiring action after it in π.

Further Improvements

Improvements for classical planning:

- Extension to conditional effects [Rintanen,Heljanko,Niemelä'06].
- Relaxed ∃-step [Wehrle&Rintanen'07].
- Parallel SAT search [Rintanen'04] [Rintanen,Heljanko,Niemelä'06].
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Extensions to non-classical planning:

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→ `https://users.aalto.fi/~rintanj1/satplan.html`

Solving Problems via Translation into SAT:

- Problem transformation is a general and important concept in computer science.
- SAT solvers are highly efficient and can be used to solve other difficult problems via transformation, even those in higher complexity classes with appropriate compilation.

Translating Classical planning into SAT:

- Classical planning problems can be translated into SAT.
- State-of-the-art improvements for this formula are based on:
  - State invariants.
  - Parallelism (∀-step, ∃-step).

Bylander'94 The Computational Complexity of Propositional STRIPS Planning.

Kautz&Selman'92 Planning as Satisfiability.

Kautz&Selman'96 Pushing the Envelope: Planning, Propositional Logic, and Stochastic Search.

Rintanen'98 A Planning Algorithm not based on Directional Search.

Rintanen'04 Evaluation Strategies for Planning as Satisfiability.

Rintanen,Heljanko,Niemelä'06 Planning as Satisfiability: Parallel Plans and Algorithms for Plan Search.

Wehrle&Rintanen'07 Planning as Satisfiability with Relaxed ∃-Step Plans.

Mattmüller&Rintanen'07 Planning for Temporally Extended Goals as Propositional Satisfiability.

Rintanen'10a Heuristic Planning with SAT: Beyond Uninformed Depth-First Search.

Rintanen'10b Heuristics for Planning with SAT.

Gocht&Balyo'17 Accelerating SAT Based Planning with Incremental SAT Solving.

Rintanen'17 Temporal Planning with Clock-Based SMT Encodings.

Behnke&Biundo'18 X and more Parallelism. Integrating LTL-Next into SAT-based Planning with Trajectory Constraints while Allowing for even more Parallelism.

Randey&Rintanen'18 Planning for Partial Observability by SAT and Graph Constraints.