

# Lecture *Hierarchical Planning*

## Chapter: *Introduction to HTN Planning*

Dr. Pascal Bercher

Institute of Artificial Intelligence,  
Ulm University, Germany

Winter Term 2018/2019

(Compiled on: February 19, 2019)

## Overview:

### 1 Introduction

- Hierarchical vs. Non-Hierarchical Planning
- Motivation for Hierarchical Planning
- Background, Vocabularies, and Conventions in Hierarchical Planning

### 2 Problem Definition

- Introduction
- Formal Problem Definition

### 3 Decomposition Trees

- Motivation
- Basic Definitions

### 4 Formalization Choices in HTN Planning



## What is Planning?

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- That's only the case for *non-hierarchical* planning and different from *hierarchical* planning!

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## What is Hierarchical Planning?

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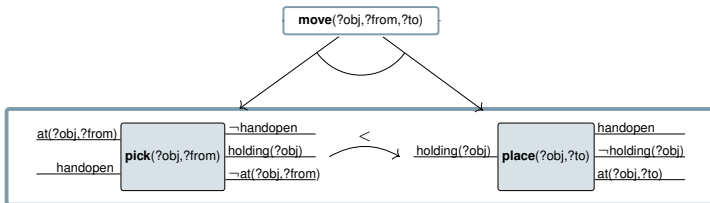
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- There is no arbitrary task insertion: to alter task networks, we need to decompose compound tasks using their pre-defined methods (see next slide).



## What is Hierarchical Planning? . . . More Precisely? Example?

The model specifies a **task hierarchy**: *compound* (or *complex*, *abstract*, *high-level*) tasks need to be decomposed into *primitive* tasks.



Goal: Find a (primitive) executable *refinement* of an initial hierarchical task network (HTN) or partial plan.

**Top:** A compound task.

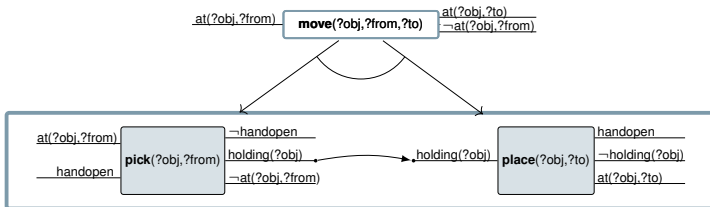
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**Top:** A compound task (with precs/effs).

**Bottom:** A partial (POCL) plan.

**Together:** A (decomposition) method.

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- *It's not about generating some goal state!* The goal is find a refinement of the initial compound task(s), *not* to satisfy some goal description.
- There is (normally) no arbitrary task insertion: To alter task networks/partial plans, we need to decompose compound tasks using their pre-defined methods. ("Task insertion" is an additional feature (actually: solution criteria!) that has to be provided/allowed in addition.)



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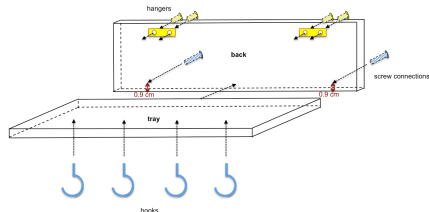
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- Use hierarchy as plan libraries (describing possible user intent) for plan recognition.
- Communicate plans on different levels of abstraction.
- Incorporate task abstraction in plan explanations.



## Motivation, Example: Do-It-Yourself (DIY) Assistant



The material:

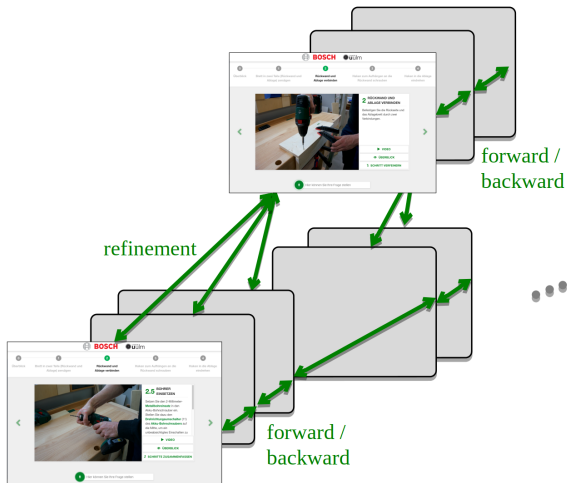
- Boards (need to be cut first)
- Electrical devices like drills and saws
- Attachments like drill bits and materials like nails

## Motivation, Example: Do-It-Yourself (DIY) Assistant, cond't

Presentation of instructions on different levels of abstraction:

Abstract Level

Detailed Level



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- One of the best-known formalizations is called *hierarchical task network* (HTN) planning, so it is often used as a synonym to hierarchical planning although the latter can be regarded as the more general expression/field.
- Since the HTN formalism can be regarded a standard and the most simplistic one, we will start with that. Later, we extend it in several directions.



## HTNs vs. HTN Problems vs. HTN Planning

- HTN is short for (*hierarchical*) *task network*.  
Thus, it is a *data structure*, not a formalism or an approach.  
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- The term “HTN planning” can still refer to either the problem class or an (HTN) planning approach (similar to classical planning).
- In the context of the HTN planning framework, we use *HTNs* as basic data structure, i.e., partially ordered tasks.

If we also use causal links there, we refer to these data structures as *partial plans* instead.



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- *Actions* known from classical planning are the same as *primitive tasks* in hierarchical planning.
- In hierarchical planning, the term *task* is used to refer to either *actions* (i.e., primitive tasks) or *abstract tasks*.





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  - Rather than plan steps being 2-tuples  $l:a$ , we have another mapping  $\alpha$  to map  $l$  to  $a$  (see later).



Most definitions (in particular: the ground HTN and TIHTN formalisms, and the decomposition tree) are taken from:

- Thomas Geier and Pascal Bercher. “On the Decidability of HTN Planning with Task Insertion”. In: *Proc. of the 22nd Int. Joint Conf. on Artificial Intelligence (IJCAI 2011)*. AAAI Press, 2011, pp. 1955–1961

Definitions of the lifted HTN and TIHTN formalisms can be found in:

**HTN** Ron Alford, Pascal Bercher, and David Aha. “Tight Bounds for HTN Planning”. In: *Proc. of the 25th Int. Conf. on Automated Planning and Scheduling (ICAPS 2015)*. AAAI Press, 2015, pp. 7–15

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## Problem Definition, Example Search Process

$\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$  with:

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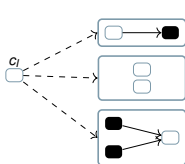
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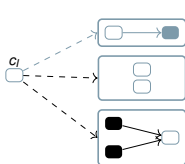
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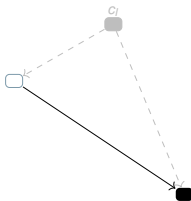
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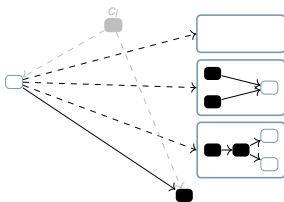
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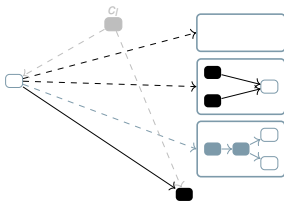
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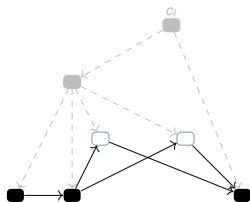
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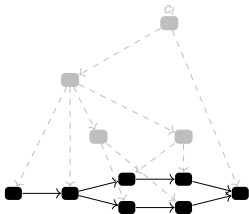
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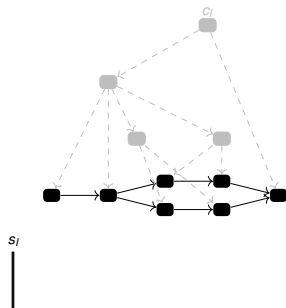
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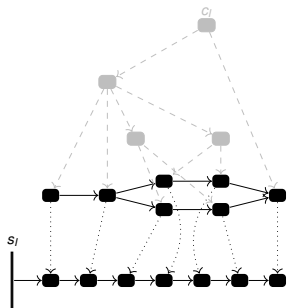
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- have an executable linearization.



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- Two task networks  $tn = (T, \prec, \alpha)$  and  $tn' = (T', \prec', \alpha')$  are called *isomorphic* (written  $tn \cong tn'$ ) if they differ solely in their task identifier symbols, i.e. there is a bijection  $\sigma : T \rightarrow T'$  so that:



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- $TN_X$  refers to the set of all task networks using only task names in  $X$ .



## Decomposition Methods

- A (*decomposition*) method  $m \in M$  is a tuple  $m = (c, tn_m)$  with a compound task  $c$  and task network  $tn_m = (T_m, \prec_m, \alpha_m)$ .



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Then, the application of  $m$  to  $tn$  results in the task network  $tn' = ((T \setminus \{t\}) \cup T_m, \prec \cup \prec_m \cup \prec_X, \alpha \cup \alpha_m)|_{(T \setminus \{t\}) \cup T_m}$  with:

$$\begin{aligned} \prec_X := & \{(t', t'') \mid (t', t) \in \prec, t'' \in T_m\} \cup \\ & \{(t'', t') \mid (t, t') \in \prec, t'' \in T_m\} \end{aligned}$$

where  $(X_1, \dots, X_n)|_Y$  restricts the sets  $X_i$  to elements in  $Y$ .





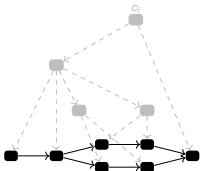
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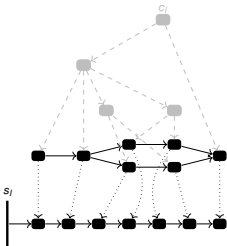
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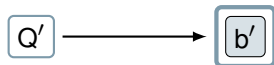
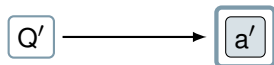
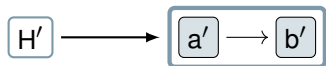
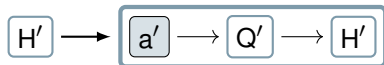
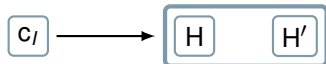
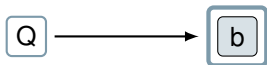
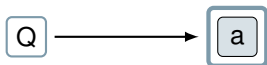
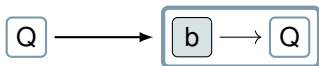
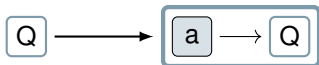
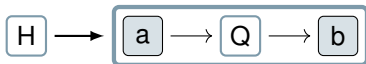
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  - $tn$  is executable, i.e.,
    - it contains only primitive tasks, and
    - the (still partially ordered) task network  $tn$  admits an executable linearization  $\bar{t}$  of its tasks.



## How to Represent Decomposition?

Consider the following decomposition methods:

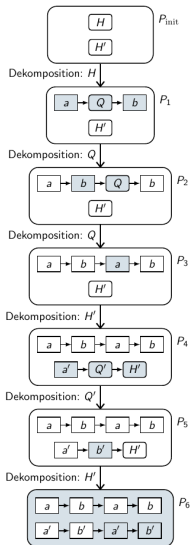


(Preconditions and effects don't matter for now.)

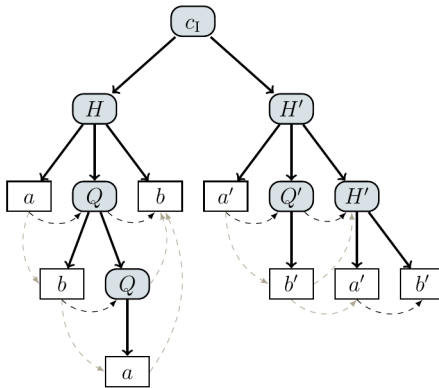


## Motivation

## Decomposition Tree: Example



Representation as a tree:



## Decomposition Tree: Definition

### Definition (Decomposition Tree)

A *decomposition tree*  $dt = (T, E, \prec, \alpha, \beta)$  is a five-tuple with the following properties:

- $(T, E)$  is a tree with task identifier symbols  $T$  (the nodes of the tree) and directed edges  $E \subseteq T \times T$  pointing towards the leaves,



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- $\beta : T' \rightarrow M \times Iso$ , with  $T' \subseteq T$ , is a function mapping each node out of a (possibly strict) superset of the inner nodes to a tuple consisting of a method  $m \in M$  and an isomorphism  $\sigma \in Iso$ ,  $Iso$  denoting the set of all isomorphisms over the task instances in  $T$ .



## Decomposition Tree: Definition, cont'd

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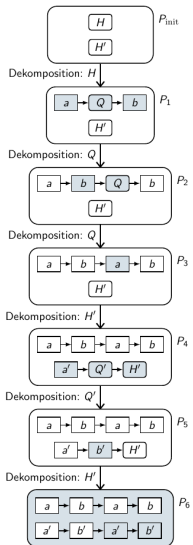
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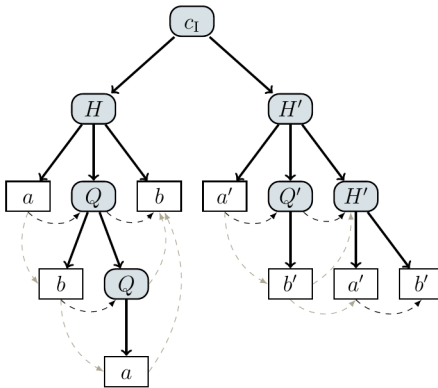


## Basic Definitions

## Decomposition Tree: Example – Are We There Yet?



Representation as a tree:



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- 3 There are no other ordering constraints than those demanded by Criterion 2 or those required by the definition of decomposition trees.



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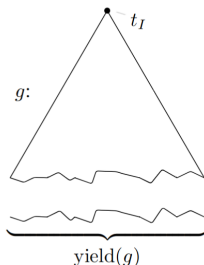


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- Then,  $yield(dt) := (T', \alpha|_{T'}, \prec|_{T'})$ .



## Properties of Decomposition Trees

### Theorem

Given a planning problem  $\mathcal{P}$ , then for any task network  $tn \in TN_{CUP}$  there exists a valid decomposition tree  $dt$  with  $yield(dt) = tn$  if and only if  $tn_I \rightarrow_{TD}^* tn$ .

*Proof:*

Straight-forward.



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Which formalization choices and extension to standard HTN planning do exist? Which impact do they have?



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- Separation into problem and domain.



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- Alternative definition of executability.
- Allowing to insert tasks.
- Adding state constraints.



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- Then,  $\mathcal{D} = (V, P, \delta, C, M)$  is the *domain* and  $\mathcal{P} = (\mathcal{D}, s_I, c_I)$  (or  $\mathcal{P} = (\mathcal{D}, s_I, tn_I)$ ) is the *problem (instance)*.



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- That way, we can also define several problems for the same domain.



## Impact of Initial Task Network

Recap:  $\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$  describes an HTN planning problem as described before.

Let  $\mathcal{P}^* = (V, P, \delta, C, M, s_I, \mathbf{tn}_I)$  be an **HTN planning problem with initial task network  $\mathbf{tn}_I$** .





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Then, a task network  $tn$  is a solution if and only if:

- There is a sequence of decomposition methods  $\bar{m}$  **that transforms  $\mathbf{tn}_I$  into  $tn$** ,
- $tn$  contains only primitive tasks, and
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## Impact of Initial Task Network, cont'd

*Theorem: Initial task networks can be compiled away.*



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*Proof:*

Let  $\mathcal{P}^* = (V, P, \delta, C, M, s_I, \mathbf{tn}_I)$  be an HTN planning problem with initial task network  $\mathbf{tn}_I$ .

Then, there is an HTN planning problem  $\mathcal{P}' = (V, P, \delta, \mathbf{C}', \mathbf{M}', s_I, \mathbf{c}_I)$  with the same set of solutions:

Let  $\mathbf{C}' := C \dot{\cup} \{c_I\}$  and  $\mathbf{M}' := M \cup \{(c_I, \mathbf{tn}_I)\}$ .

Identical solution set is obvious.



## Impact of Goal Description

Recap:  $\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$  describes an HTN planning problem as described before.

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- **the task sequence  $\bar{t}$  generates a goal state  $\mathbf{s} \supseteq \mathbf{g}$ .**



## Impact of Goal Description, cont'd

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*Proof:*

Let  $\mathcal{P}^* = (V, P, \delta, C, M, s_I, c_I, \mathbf{g})$  be an HTN planning problem with goal description.

Then, there is an HTN planning problem  $\mathcal{P}' = (V, \mathbf{P}', \delta', C, M, s_I, \mathbf{tn}_I)$  with the same set of solutions:

Here,  $\mathbf{tn}_I$  contains two tasks:  $c_I$  followed by a new primitive task  $p$  with no effects and  $g$  as precondition,  $\delta(p) = (g, \emptyset, \emptyset)$ .



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Then, the initial task network in  $\mathcal{P}'$  can be compiled away as before.

Identical solution set is obvious.





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- Plan verification becomes easier (see later chapter).
- For this criterion, we must allow ordering insertion, as otherwise solutions with the demanded properties might not exist.



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- Task insertion plus goal description fully subsumes classical planning (while allowing task hierarchies as well).
- Task insertion makes the modeling process easier: certain parts can be left to the planner.
- Task insertion makes the problem computationally easier (can be exploited for heuristics).



## Problem Definition

In *HTN planning with task insertion*, *TIHTN planning*, tasks may be added arbitrarily to task networks (not just via decomposition):

Let  $\mathcal{P}^* = (V, P, \delta, C, M, s_I, c_I)$  be a **TIHTN planning problem**.



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Then, a task network  $tn$  is a solution if and only if:

- There is a sequence of decomposition methods  $\bar{m}$  **and task insertions** that transforms  $c_I$  into  $tn$ ,
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- $tn \supseteq tn'$  **contains all tasks and orderings of  $tn'$** ,  
(Note: allowing  $\prec \supseteq \prec'$  would imply that we allow ordering insertion, which would, similar to HTN planning, be required if we demand all linearizations to be executable.)
- $tn$  contains only primitive tasks, and
- the (still partially ordered) task network  $tn$  admits an executable linearization  $\bar{t}$  of its tasks.



## Problem Formalization

State constraints have been introduced in the HTN formalization by Erol et al. (1994):

- $(l, t)$ , the literal  $l$  holds immediately before task  $t$ .
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In case  $t$ , resp.  $t'$ , are compound, a constraint  $(l, t)$  is, upon decomposition, translated to  $(l, first[t_1, \dots, t_n])$ , where the  $t_i$  are all sub tasks of  $t$ . ( $(t, l)$  and  $(t, l, t')$  are handled analogously.)



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*No compilation known yet.*





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- There are various different hierarchical planning formalisms (some of them covered later) with different theoretical properties.
- HTN planning is *the* standard hierarchical planning formalism.
- Also for HTN planning there are various formalization choices with differing impact on theoretical properties.

