Lecture *Hierarchical Planning*

Chapter:
Expressivity Analysis of Planning Formalisms

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Overview:

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$\rightarrow$ Expressivity Analysis: Which *structural* properties may solutions have?

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The *language* of a grammar, $L(G)$, is the set of words that can be obtained from G's start symbol by applying a sequence of G's production rules.

Formal Grammars, Example

Let $G = (\Gamma, \Sigma, R, S)$ with $\Gamma = \{S, A, B\}$, $\Sigma = \{a, b\}$, and R given by:

- $S \rightarrow aA$
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$L(G) = \{a^n b^m \mid n, m \geq 1\}$

Chomsky Hierarchy

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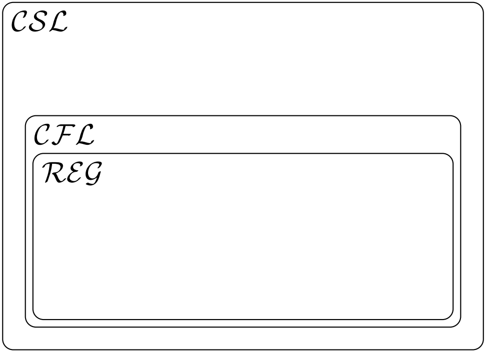
- Type 3 Regular grammars.

Expressivity via Comparison to Formal Languages



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Properties:

- All finite languages are regular. (But not the other way round.)
- There is an equivalent definition based on DFAs.
- Do you know "regular expressions"?

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- Given a context-free grammar, deciding whether it describes a regular language is undecidable. (Cf. p.281 of John E. Hopcroft and Jeffrey D. Ullman. *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley, 1979)

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  - S is not mentioned in any right-hand side.

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- No restrictions on the production rules.

Expressivity: Example

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- Is is also regular?

Recap: Standard Decision Problems for Formal Languages

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- The *language classification problem*: Given a set of words (i.e., a language), is there a grammar with certain properties that produces it?

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Planning: Relationship to Formal Languages

Planning Problem $\longrightarrow$ Set of Solutions

Formal Grammar $\longrightarrow$ Set of Words (Language)

Semantical Correspondence:

- Each planning problem can be interpreted as a compact representation of its solutions.
- Similarly, each formal grammar is a compact representation of its set of words, i.e., its language.

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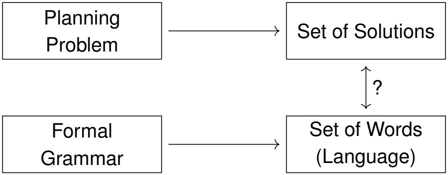

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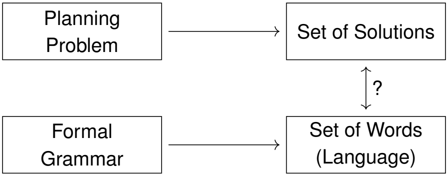

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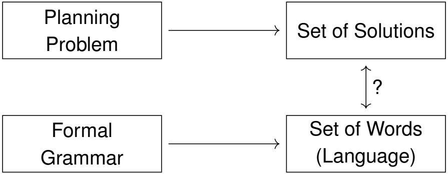

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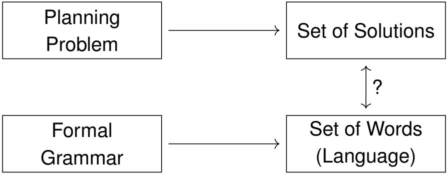

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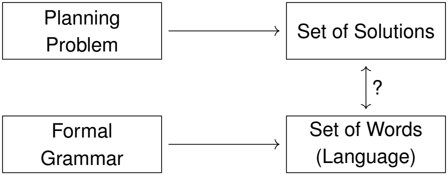

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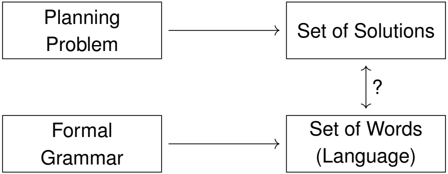

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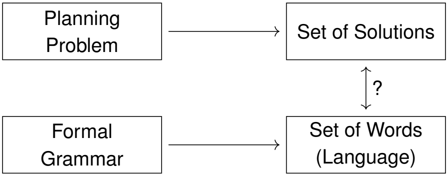

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- Analysis also works for TIHTN planning or non-hierarchical planning.

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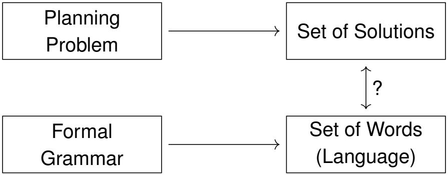

Further reading, including all of the next results:

- Daniel Höller et al. "Language Classification of Hierarchical Planning Problems". In: *Proc. of the 21st Europ. Conf. on Artificial Intelligence (ECAI 2014)*. IOS Press, 2014, pp. 447–452. DOI: 10.3233/978-1-61499-419-0-447
- Daniel Höller et al. "Assessing the Expressivity of Planning Formalisms through the Comparison to Formal Languages". In: *Proc. of the 26th Int. Conf. on Automated Planning and Scheduling (ICAPS 2016)*. AAAI Press, 2016, pp. 158–165

A Closer Look to the Relationship of Planning to Formal Grammars

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The *language intersection problem* and the *language classification problem* are interesting (and useful) from a theoretical point of view, but there is no immediate correspondence to standard "planning questions".

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  - For standard HTN planning, *every executability witness* of any solution is in the language.
  - For HTN planning with *all executability semantics*, every linearization of any solution is in the language.

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- Example: $\mathcal{STRIPS} = \{L(\mathcal{P}) \mid \mathcal{P}$ is a STRIPS planning problem$\}$

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- Let $\mathcal{P}$ be a STRIPS planning planning problem with empty goal description.
- The set of solutions of this *EXE (executablity) problem* is exactly the set of executable action sequences.
- With $\mathcal{EXE}$ we refer to the language of the respective problem class.
- Because of the missing goal description, EXE problems are less expressive than the regular languages.

The *EXE* “Planning Problem”, cont’d

Theorem

$\mathcal{EXE} \subsetneq \mathcal{REG}$

Proof:

The *EXE* "Planning Problem", cont'd

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$\mathcal{EXE} \subsetneq \mathcal{REG}$

Proof:

1. Show for all $L \in \mathcal{EXE}$ that $L \in \mathcal{REG}$. How?

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1. Show for all $L \in \mathcal{EXE}$ that $L \in \mathcal{REG}$. How? Construct an automaton.
2. Provide a language $L \in \mathcal{REG}$ with $L \notin \mathcal{EXE}$. How? Exploit an important property: If some plan is executable, than every prefix is as well (due to the missing goal description).

STRIPS

Theorem

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Proof:

STRIPS

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2. Provide a language $L \in \mathcal{REG}$ with $L \notin \mathcal{STRIPS}$. How? Again, provide a finite language that cannot be expressed as a STRIPS planning problem.

STRIPS, cont'd

For the second step in the previous proof, exploit:

Theorem

Let $s \in S$ be a state and $a \in A$ an action. If a is applicable in s' (resulting from applying a in s), then a is applicable arbitrarily often.

STRIPS, cont'd

For the second step in the previous proof, exploit:

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Proof:
Exercise (just show it directly via playing with preconditions and effects).

STRIPS with Conditional Effects

Theorem

The language of STRIPS problems with conditional effects, $\mathcal{STRIPS}\text{–}\mathcal{CE}$, is equivalent to the regular languages, $\mathcal{REG}$.

Proof:

STRIPS with Conditional Effects

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The language of STRIPS problems with conditional effects, $\mathcal{STRIPS{-}CE}$, is equivalent to the regular languages, $\mathcal{REG}$.

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1. For every SCE planning problem, there is an equivalent regular language.

STRIPS with Conditional Effects

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The language of STRIPS problems with conditional effects, $\mathcal{STRIPS}{-}\mathcal{CE}$, is equivalent to the regular languages, $\mathcal{REG}$.

Proof:

1. For every SCE planning problem, there is an equivalent regular language.
2. For every regular language, there is a SCE problem generating it.

STRIPS with Conditional Effects, cont'd

- Let $\mathcal{P} = (V, A, s_I, g)$ be a planning problem.

STRIPS with Conditional Effects, cont'd

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  - Every goal state $s \supseteq g$ is included in F.

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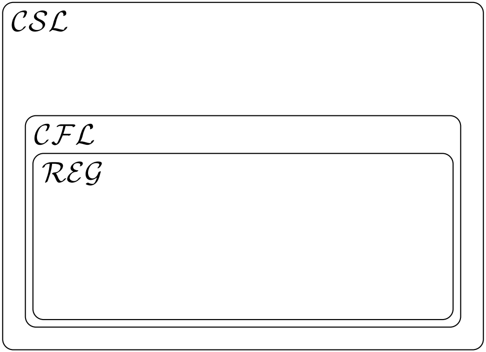
$$
\begin{aligned}
\forall a \in A : \mathit{prec}(a) &= \emptyset \\
\mathit{add}(a) &= \{(\{s\} \to \{s'\} \cup G') \mid d(s,a) = s'\} \\
\text{with } G' &= \begin{cases} \{g\}, & \text{if } s' \in F \\ \emptyset, & \text{else} \end{cases} \\
\mathit{del}(a) &= \{(\emptyset \to V)\}
\end{aligned}
$$

Expressivity via Comparison to Formal Languages



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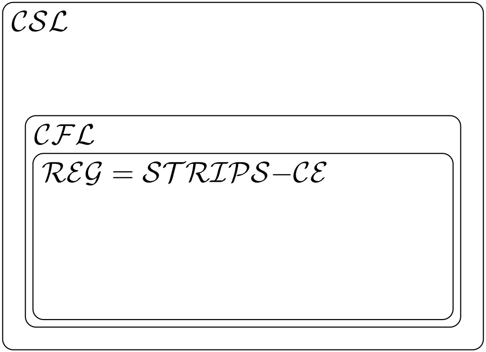
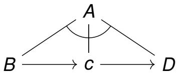


Totally Ordered HTN Planning Problems

- Decomposition in totally ordered HTN planning problems is similar to rule application in context-free grammars.



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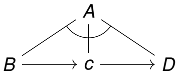

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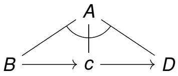

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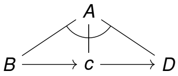

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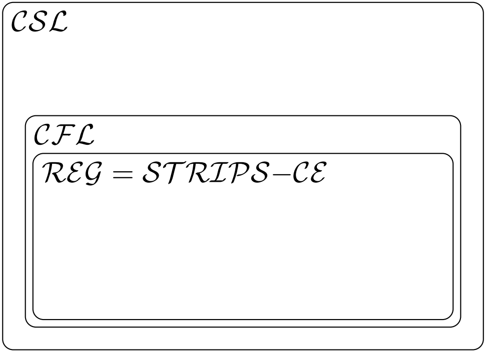
- The encoding of (totally ordered) HTN decomposition as (context-free) grammar rules and vice versa is straightforward.
- $\mathcal{HTN}{-}\mathcal{ORD} \supseteq \mathcal{CFL}$ is trivial, since no states are required.
- Constraints introduced by preconditions and effects can be treated via intersection with a regular language:

  Remember that the intersection of any context-free language with any regular language is still context-free. Thus, we can intersect the language representing the hierarchy (which is context-free) with one of the regular languages $\mathcal{EXE}$ or $\mathcal{STRIPS}$ (do we feature a goal description?) to show $\mathcal{HTN}{-}\mathcal{ORD} \subseteq \mathcal{CFL}$.

Expressivity via Comparison to Formal Languages



Expressivity via Comparison to Formal Languages

$\mathcal{CSL}$

$\mathcal{CFL} = \mathcal{HTN}\text{-}\mathcal{ORD}$

$\mathcal{REG} = \mathcal{STRIPS}\text{-}\mathcal{CE}$

Acyclic HTN Problems

- Informally/intuitively, *acyclic HTN/TIHTN problems* are problems where no recursion is possible.
- There are many equivalent formal definitions, some of them will be covered later. For instance: For every task network that is reachable via decomposition from the initial task network holds: Let *dt* be its decomposition tree. Then, no path from its root node to any of its leafs contains the same task more than once.

TIHTN and Acyclic HTN Problems

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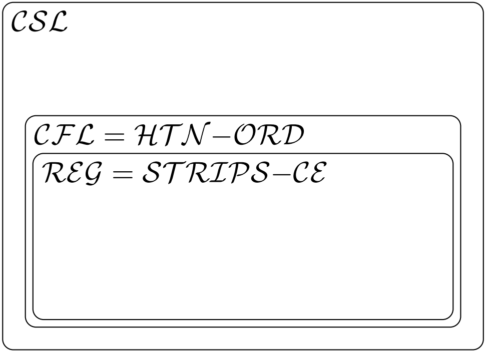
These results rely on the presence of goal descriptions! More details in the exercises.

Expressivity via Comparison to Formal Languages



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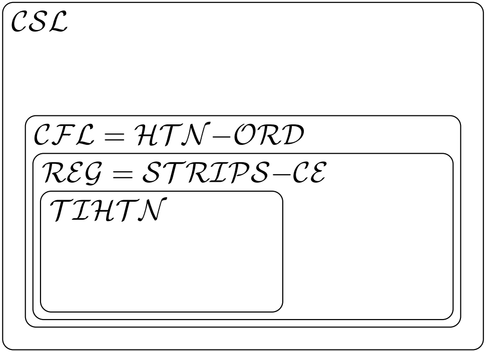

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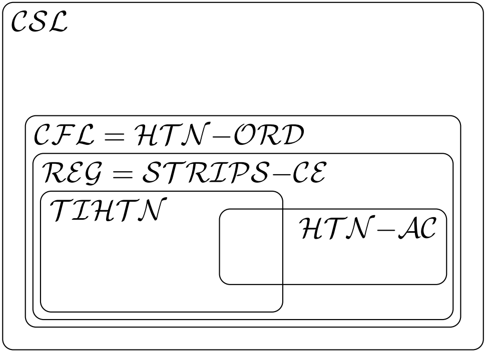

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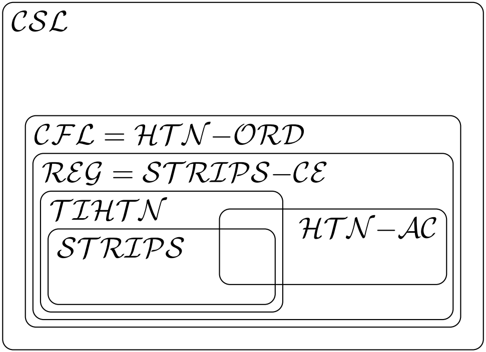

Noop HTN Planning Problems

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Noop HTN Planning Problems

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- First class we look at:

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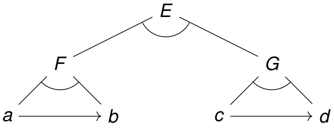
- Can a partially ordered method be transformed into a set of totally ordered methods?

Noop HTN Planning Problems, cont'd I

HTN

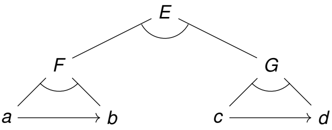


Grammar

$$E \rightarrow FG \quad F \rightarrow ab$$
$$E \rightarrow GF \quad G \rightarrow cd$$

Noop HTN Planning Problems, cont'd I

HTN



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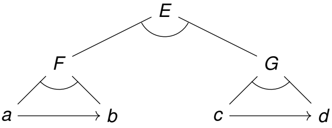

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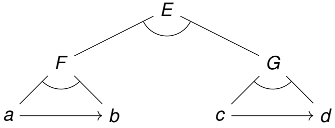


Grammar

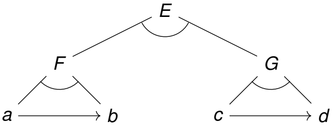
$E \rightarrow FG \quad F \rightarrow ab$

$E \rightarrow GF \quad G \rightarrow cd$

Word 1 *cdab* ✓ Word 2 *acbd*

Noop HTN Planning Problems, cont'd I

HTN



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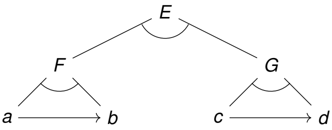

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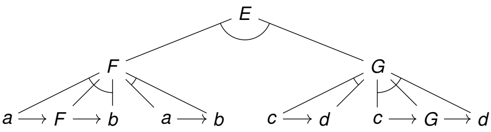
$\{abcd\} \cup \{cdab\}$

Noop HTN Planning Problems, cont'd II

- The HTN depicted below generates the language $a^n b^n || c^m d^m$.



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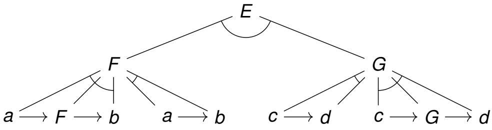

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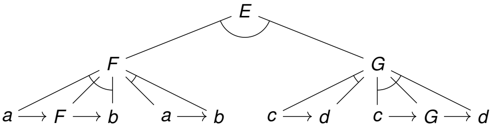
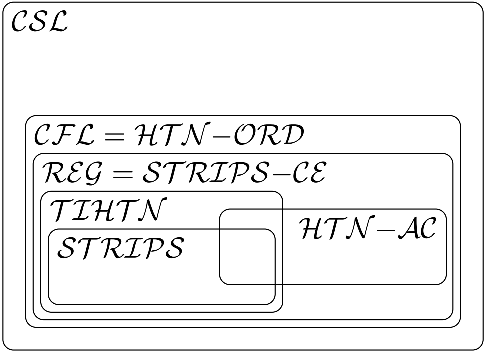
$\rightarrow$ $\mathcal{CFL} \subsetneq \mathcal{HTN}\text{–}\mathcal{NOOP}$



Expressivity via Comparison to Formal Languages



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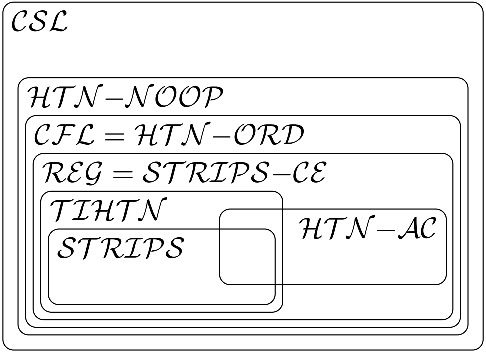

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$\rightarrow$ These results are just mentioned for the sake of completeness. Proofs are omitted.

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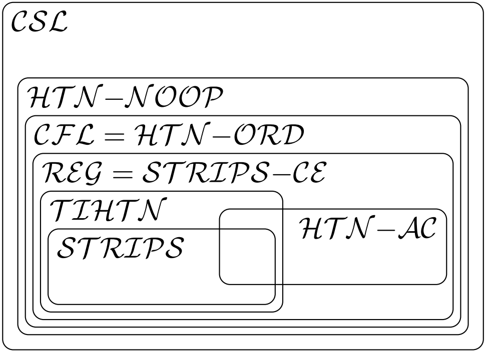

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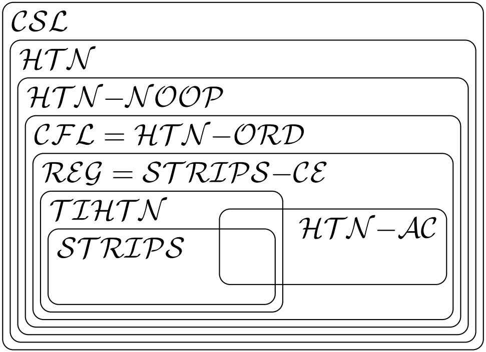

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- Even higher language features, e.g., functions.

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