## Lecture Hierarchical Planning

## Chapter: Complexity Results for Plan Verification

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Introduction			

### **Overview:**

- 1 Introduction
- 2 Recap on Complexity Theory
- 3 Plan Verification
- 4 Verify Non-hierarchical Plans
- 5 Verify Hierarchical Plans

## 6 Summary



Introduction ●O			

Complexity analysis studies the computational hardness of a decision problem. In this lecture we study:



Introduction ●○			

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The plan existence problem:



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- The plan existence problem: How hard is it to decide whether a problem *P* has a solution?
- The plan verification problem:



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- The plan existence problem: How hard is it to decide whether a problem P has a solution?
- The plan verification problem: How hard is it to decide whether a given plan is actually a solution?



Introduction			



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Benefits of complexity studies:

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  Example: heuristics! (Most of them exploit special cases that can be decided in P.)





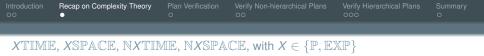
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- We can identify special cases to be exploited by algorithms. Example: heuristics! (Most of them exploit special cases that can be decided in P.)
- Insights may also allow for compilation techniques.
- Last, but not-at-all least: they help understanding the problem! (Understanding the problem should always be the first step.)





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- Note: PSPACE = NPSPACE (holds also for higher classes).





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  - Empty or primitive initial task network.
  - Totally ordered methods/initial task network.



Introduction		Verify Non-hierarchical Plans ●O	

#### Verification of Classical Solutions

#### Theorem

Let  $\mathcal P$  be a STRIPS planning problem and  $\bar a$  an action sequence. Then, deciding whether  $\bar a$  is a solution for  $\mathcal P$  is



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#### Proof:

Execute the plan and check whether every action can be applied in the respective state and whether a goal is produced. ( $\rightarrow$  Linear effort.)



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### Proof:

Membership: guess and verify.

Hardness: Reduction from CNF Sat (proof idea via black board).



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# Corollary



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Let *tn* be a primitive task network and  $\mathcal{P}$  an HTN or TIHTN planning problem. Then, deciding whether *tn* is a solution is  $\mathbb{NP}$ -hard.

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What about membership? Easy if methods are non-empty, tricky (but possible) otherwise.



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Verificat	tion of HTN Plans			

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  - $\rightarrow$  Then, checking executability is in  $\mathbb P,$  right?
    - No! This is only the case if we know the *labels/task ids* rather than just the actions.
    - Otherwise, we have to check whether there is a refinement of the task network's ordering constraint leading to the witness, which is again NP-hard.





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Deciding whether a (primitive) task network is executable (in the sense given above) is in  $\mathbb{P}.$ 

# Proof: Black board.





- So, given a task network with *all executability semantics*, plan verification can be decided in P, right?
- No! We still need to check the refinement criterion. This is NP-hard, however. (Reduction from Vertex Cover, maybe later.)





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Deciding whether a (primitive) task network is a solution is, even for *all* executability semantics,  $\mathbb{NP}$ -complete.





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- The hardness of verifying partially ordered plans depends on whether an executable linearization needs to exist or whether all of them need to be executable.
- Verifying hierarchical plans is often harder, because we also need to check the refinement criterion.
- Complexity results give raise to specialized algorithms, to heuristics, and to translations to other problem classes.

