

Chapter:

Solving Hierarchical Planning Problems via SAT

Gregor Behnke

Institute of Artificial Intelligence,
Ulm University, Germany

Winter Term 2018/2019
(Compiled on: February 20, 2019)



Overview:

- 1 Theoretical Background
 - Bridging the Gap between NP and PSPACE
 - Bridging the Gap between NP and Undecidability
- 2 What are we looking for?
- 3 Compactifying Decomposition Trees
- 4 SAT Encoding
 - Decomposition
 - Executability
- 5 Evaluation



HTN Planning via SAT

In lecture 04 (Solving (Non-Hierarchical) Planning Problems via SAT) we have seen how classical planning problems can be solved via a translation into SAT.

Can this also be done for HTN planning?



Issues with HTN Planning via SAT

Reminder: For a planning problem \mathcal{P} create a CNF formula \mathcal{F} that is satisfiable iff \mathcal{P} has a solution.

(Potential) Issues:

- HTN planning is undecidable, i.e. there cannot be such a formula \mathcal{F} .
- Even if we find a way, how do we represent decomposition?




Theoretical Background ●○○○ What are we looking for? ○○ Compactifying Decomposition Trees ○○○○ SAT Encoding ○○○○○○○○○○ Evaluation ○ Summary ○○

Bridging the Gap between NP and PSPACE

Idea for Transforming Classical Planning

- PLANEX is PSPACE
- PLANEX “is” NP-“complete” for ℓ -length bounded planning if ℓ is encoded unary.
- For full PLANEX: theoretical limit $2^{|V|}$.
- Start with a small ℓ and increase until a solution is found.

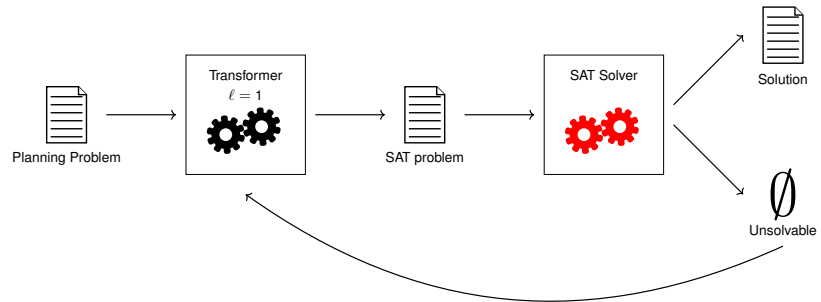



Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 5 / 30

Theoretical Background ●○○○ What are we looking for? ○○ Compactifying Decomposition Trees ○○○○ SAT Encoding ○○○○○○○○○○ Evaluation ○ Summary ○○

Bridging the Gap between NP and PSPACE

Bound Iteration

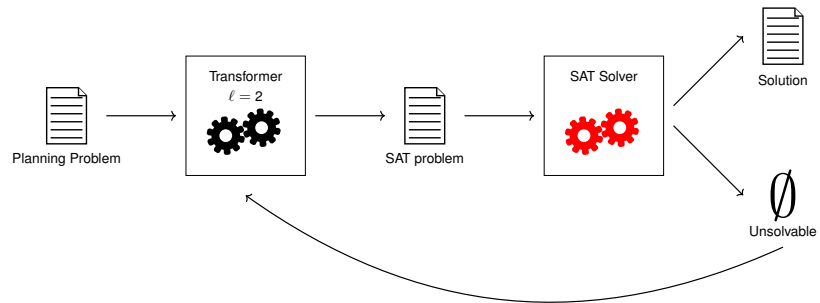




Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 6 / 30

Theoretical Background ●○○○ What are we looking for? ○○ Compactifying Decomposition Trees ○○○○ SAT Encoding ○○○○○○○○○○ Evaluation ○ Summary ○○

Bridging the Gap between NP and PSPACE

Bound Iteration

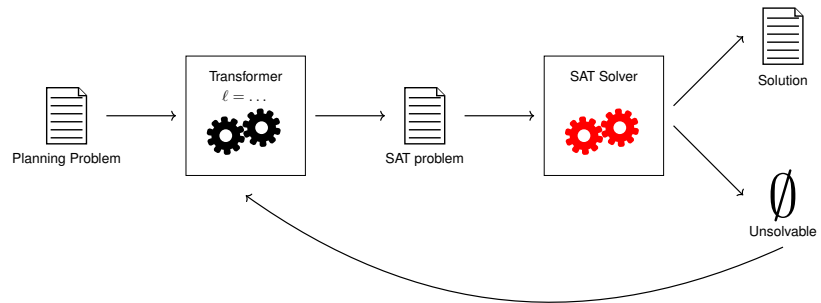




Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 6 / 30

Theoretical Background ●○○○ What are we looking for? ○○ Compactifying Decomposition Trees ○○○○ SAT Encoding ○○○○○○○○○○ Evaluation ○ Summary ○○

Bridging the Gap between NP and PSPACE

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 6 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○○ ○○ ○○○○ ○○○○○○○○○○ ○ ○○

Bridging the Gap between NP and PSPACE

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 6 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○○ ○○ ○○○○ ○○○○○○○○○○ ○ ○○

Bridging the Gap between NP and Undecidability

Idea for Transforming HTN Planning

Why not do the same for HTN planning?

- Start with a small length bound ℓ and increase until a solution is found.
- For full PLANEX: upper bound depends on the problem!
 - acyclic: maximum decomposable length
 - totally-ordered: maximum decomposable length with depth $(2^{|\mathcal{V}|})^2 |\mathcal{C}|$
 - regular: exercise
 - tail-recursive: more complex, use stratification
 - general: ∞
- For general HTNs we can only construct algorithm that terminates on success but not (always) on failure.
- “Is” PLANEX NP-“complete” for ℓ -length bounded HTN planning if ℓ is encoded unary?

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 7 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○○ ○○ ○○○○ ○○○○○○○○○○ ○ ○○

Bridging the Gap between NP and Undecidability

Idea for Transforming HTN Planning

Is *plan length* a good bound for HTN planning?
 Not really.
 A length bound follows easily from a depth bound, but not the other way around.
 We use *decomposition depth* instead.

“Is” PLANEX NP-“complete” for K -depth bounded HTN planning if K is encoded unary?

Issue? We loose optimality!

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 8 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○○ ○○ ○○○○ ○○○○○○○○○○ ○ ○○

Bridging the Gap between NP and Undecidability

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 9 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○● ○○ ○○○○ ○○○○○○○○○ ○ ○ ○○

Bridging the Gap between NP and Undecidability

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 9 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○● ○○ ○○○○ ○○○○○○○○○ ○ ○ ○○

Bridging the Gap between NP and Undecidability

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 9 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○● ○○ ○○○○ ○○○○○○○○○ ○ ○ ○○

Bridging the Gap between NP and Undecidability

Bound Iteration

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 9 / 30

Theoretical Background What are we looking for? Compactifying Decomposition Trees SAT Encoding Evaluation Summary

○○○○○ ●○○ ○○○○ ○○○○○○○○○ ○ ○ ○○

Our Objective

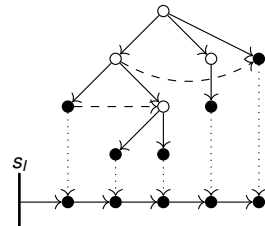
Given an HTN planning problem \mathcal{P} and a depth bound K ,
 construct a CNF formula \mathcal{F} that is satisfiable iff
 \mathcal{P} has a solution whose decomposition depth is $\leq K$

A satisfying valuation β of \mathcal{F} should represent a solution to \mathcal{P} .

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 10 / 30

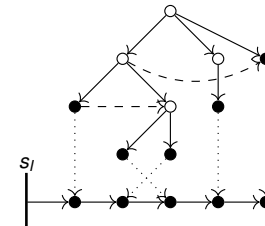
Solutions in HTN Planning

What is the *solution* to an HTN planning problem?



Solutions in HTN Planning

What is the *solution* to an HTN planning problem?

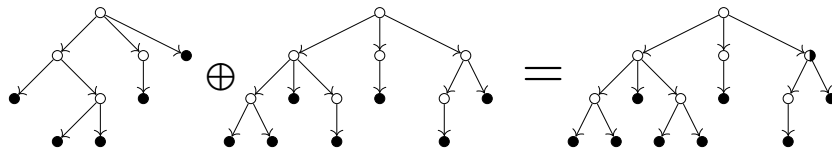


β should represent a *Decomposition Tree*.



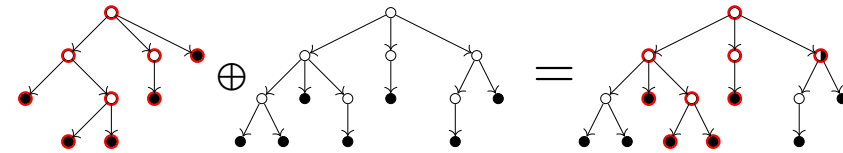
Representing Decomposition Trees Compactly

- There are doubly-exponentially many trees of depth $\leq K$
 - We need a compact representation of all possible Decomposition Trees of depth $\leq K$
- ⇒ Construct a super-tree of all possible Decomposition Trees



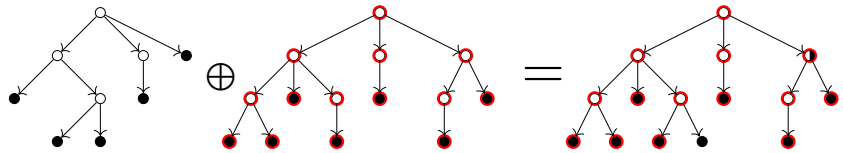
Representing Decomposition Trees Compactly

- There are doubly-exponentially many trees of depth $\leq K$
 - We need a compact representation of all possible Decomposition Trees of depth $\leq K$
- ⇒ Construct a super-tree of all possible Decomposition Trees



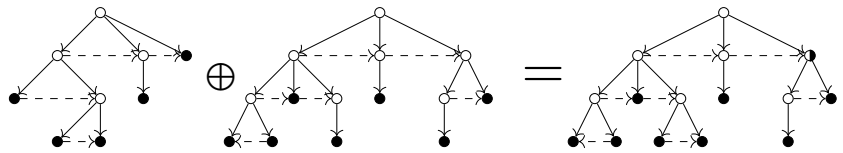
Representing Decomposition Trees Compactly

- There are doubly-exponentially many trees of depth $\leq K$
 - We need a compact representation of all possible Decomposition Trees of depth $\leq K$
- ⇒ Construct a super-tree of all possible Decomposition Trees



Representing Decomposition Trees Compactly

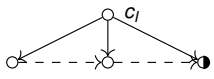
- There are doubly-exponentially many trees of depth $\leq K$
 - We need a compact representation of all possible Decomposition Trees of depth $\leq K$
- ⇒ Construct a super-tree of all possible Decomposition Trees



First, we assume *totally-ordered* planning problems.

Constructing Path Decomposition Trees

- It's infeasible to compute **all** Decomposition Trees and **then** to merge them
- ⇒ Compute super-tree by step-wise local expansion

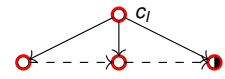


$c_l \rightarrow ABC$ and $c_l \rightarrow CBp$ and $c_l \rightarrow Ar$
 $\{A, C\}$ $\{B\}$ $\{C, p, r\}$



Constructing Path Decomposition Trees

- It's infeasible to compute **all** Decomposition Trees and **then** to merge them
- ⇒ Compute super-tree by step-wise local expansion

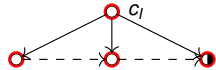


$c_l \rightarrow ABC$ and $c_l \rightarrow CBp$ and $c_l \rightarrow Ar$
 $\{A, C\}$ $\{B\}$ $\{C, p, r\}$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them
 ⇒ Compute super-tree by step-wise local expansion



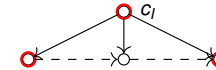
$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them
 ⇒ Compute super-tree by step-wise local expansion



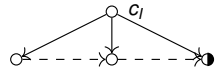
$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them
 ⇒ Compute super-tree by step-wise local expansion



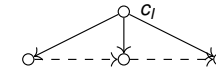
$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them
 ⇒ Compute super-tree by step-wise local expansion



$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

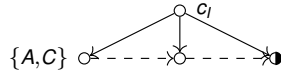
$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them

⇒ Compute super-tree by step-wise local expansion



$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

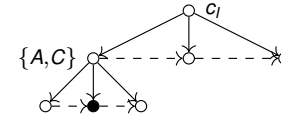
$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them

⇒ Compute super-tree by step-wise local expansion



$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

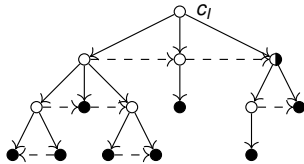
$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



Constructing Path Decomposition Trees

It's infeasible to compute **all** Decomposition Trees and **then** to merge them

⇒ Compute super-tree by step-wise local expansion



$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$



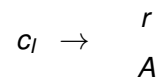
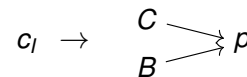
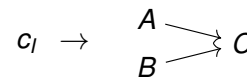
Path Decomposition Trees with Partial Order

$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$

What about methods containing *partial* order?

- 1 Guess a linearization and check order later (tree encoding)
- 2 Merge task-labeled DAGs instead of task sequences



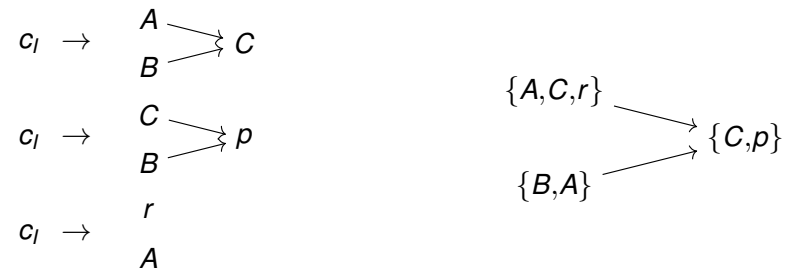
Path Decomposition Trees with Partial Order

$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$

What about methods containing *partial* order?

- 1 Guess a linearization and check order later (tree encoding)
- 2 Merge task-labeled DAGs instead of task sequences



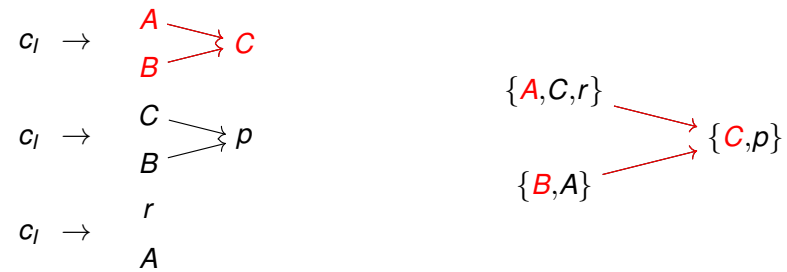
Path Decomposition Trees with Partial Order

$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$

What about methods containing *partial* order?

- 1 Guess a linearization and check order later (tree encoding)
- 2 Merge task-labeled DAGs instead of task sequences



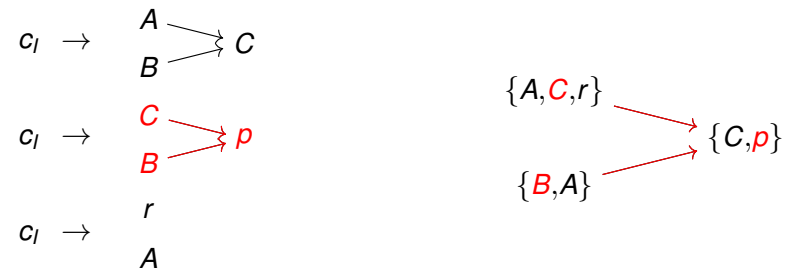
Path Decomposition Trees with Partial Order

$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$

What about methods containing *partial* order?

- 1 Guess a linearization and check order later (tree encoding)
- 2 Merge task-labeled DAGs instead of task sequences



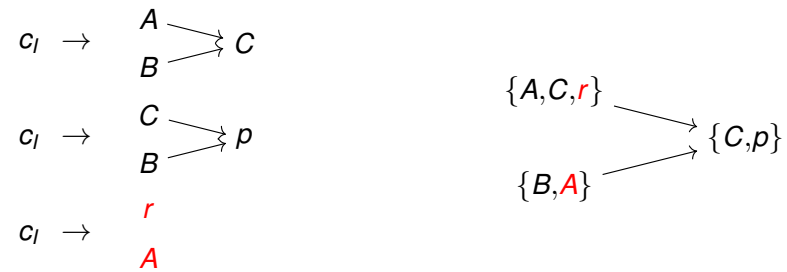
Path Decomposition Trees with Partial Order

$$c_l \rightarrow ABC \text{ and } c_l \rightarrow CBp \text{ and } c_l \rightarrow Ar$$

$$\{A, C\} \quad \{B\} \quad \{C, p, r\}$$

What about methods containing *partial* order?

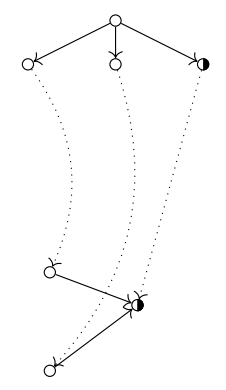
- 1 Guess a linearization and check order later (tree encoding)
- 2 Merge task-labeled DAGs instead of task sequences



Merging Decomposition Methods

Merging Methods

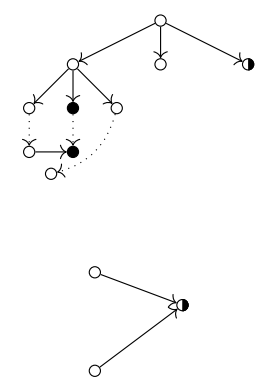
Given a family G_i of vertex-labeled transitively closed DAGs.
 Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .



Merging Decomposition Methods

Merging Methods

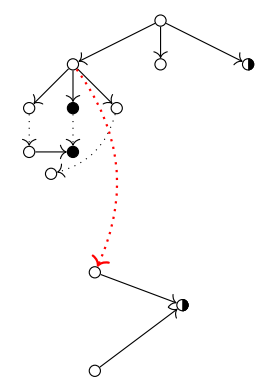
Given a family G_i of vertex-labeled transitively closed DAGs.
 Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .



Merging Decomposition Methods

Merging Methods

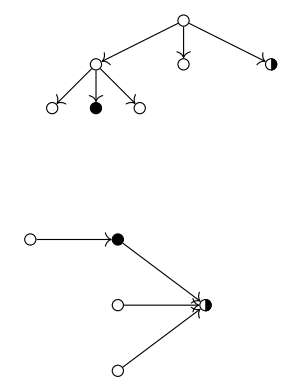
Given a family G_i of vertex-labeled transitively closed DAGs.
 Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .



Merging Decomposition Methods

Merging Methods

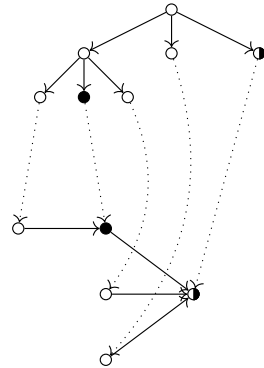
Given a family G_i of vertex-labeled transitively closed DAGs.
 Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .



Merging Decomposition Methods

Merging Methods

Given a family G_i of vertex-labeled transitively closed DAGs.
Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .

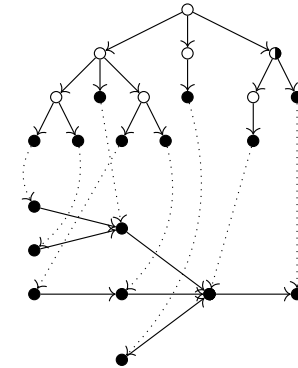


Merging Decomposition Methods

Merging Methods

Given a family G_i of vertex-labeled transitively closed DAGs.
Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .

- Resulting ordering of leaves is called *Solution Order Graph S*
- Any** task network derivable via decomposition is an induced subgraph of S
- When checking executability, we only have to consider the ordering in S , which is fixed – independent of selected methods



Minimising the SOG

Merging Methods

Given a family G_i of vertex-labeled transitively closed DAGs.
Find a vertex set-labeled graph G^* s.t. all G_i are induced subgraphs of G^* .

Difficult question: How does an optimal PDT look like?

- Fewer leaves?
- Fewer tasks per leaf?
- Fewer tasks per inner node?
- Fewer edges in the Solution Order Graph?

⇒ Optimising number of children does not lead to global minimum!

Theorem

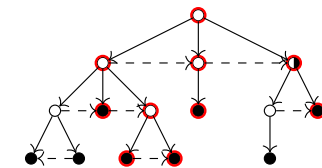
Even minimising the size of G^* is NP-complete.



Decomposition

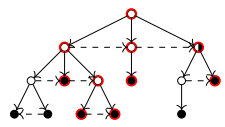
What are PDTs Good for?

- A PDT contains **every** Decomposition Tree of height $\leq K$ as a rooted sub-tree
- Let the valuation β of \mathcal{F} describe such a tree
- The formula then asserts that it is a valid DT
- Two types of decision variables:
 - a^v – node v is part of the DT and is labeled with the task a
 - m^v – method m is applied to the task in node v



Encoding – Overview

- Two types of decision variables:
 - a^v – node v is part of the DT and is labeled with the task a
 - m^v – method m is applied to the task in node v
- We then have to ensure the following properties for each node v :
 - 1 if a^v is true and $a \in C$, then $\bigvee_{m=(a,tn) \in M} m^v$
 - 2 if a^v is true and $a \in P$ or all a^v are false, then $\bigwedge_{m \in M} \neg m^v$
 - 3 if m^v is true, the children assigned by the PDT contain the correct tasks
 - 4 if all m^v are false, all $a^{v'}$ are false for all children v'
 - 5 at most one a^v and m^v is true *Really?*
 - 6 c_j^i is true



Encoding PDTs

$$\mathcal{F} = \mathcal{F}(r) \wedge c_j^i \wedge \mathcal{F}_{exe} \tag{6}$$

$$\mathcal{F}(v) = \mathbb{M}(\{t^v \mid t \in \alpha(v)\}) \wedge \mathbb{M}(\{m^v \mid M(\alpha(v) \cap C)\}) \wedge selectedMethod(v) \wedge applyMethod(v) \wedge nonePresent(v) \wedge inheritPrimitive(v) \tag{5}$$

$$selectedMethod(v) = \left[\bigwedge_{m \in M(\alpha(v) \cap C)} (m^v \rightarrow t^v) \right] \wedge \left[\bigwedge_{t \in \alpha(v) \cap C} \left(t^v \rightarrow \bigvee_{m \in M(t)} m^v \right) \right] \tag{1\&2\&4}$$

$$applyMethod(v) = \bigwedge_{m=(t,tn) \in M(\alpha(v))} \left[m^v \rightarrow \left(\bigwedge_{i=1}^{|tn|} t_{tn,i}^{v_i} \wedge \bigwedge_{v_i \in E(v) \setminus \{v_1, \dots, v_{|tn|}\}} \bigwedge_{t_i \in \alpha(v)} \neg t_i^{v_i} \right) \right] \tag{3}$$

$$nonePresent(v) = \left(\bigwedge_{t \in \alpha(v)} \neg t^v \right) \rightarrow \left(\bigwedge_{v_i \in E(v)} \bigwedge_{t \in \mathcal{C} \cup P} \neg t^{v_i} \right) \wedge \bigwedge_{t \in \alpha(v) \cap P} \left(t^v \rightarrow \bigwedge_{v_i \in E(v)} \bigwedge_{t \in \mathcal{C} \cup P} \neg t^{v_i} \right) \tag{4\&2}$$

$$inheritPrimitive(v) = \bigwedge_{p \in \alpha(v) \cap P} \left[p^v \rightarrow \left(p^{v_1} \wedge \bigwedge_{v_i \in E(v) \setminus \{v_1\}} \bigwedge_{k \in \alpha(v)} \neg k^{v_i} \right) \right]$$

$\alpha(v)$ is the set of labels of each vertex of the PDT.
 $E(v)$ are the children of v in the PDT.
 For every method $m = (c, tn)$, let v_i be the child to which the task $t_{tn,i}$ is assigned.
 $\mathbb{M}(A)$ is any encoding of the at-most-one constraint over the set of decision variables A .



What now?

We have a formula \mathcal{F} that is satisfiable iff it represents a valid Decomposition Tree T .



What now?

We have a formula \mathcal{F} that is satisfiable iff it represents a valid Decomposition Tree T .

To ensure that it is a solution, we have to check whether the leaves of T are executable in s_j in a valid linearization.



Theoretical Background 00000 What are we looking for? 00 Compactifying Decomposition Trees 00000 **SAT Encoding** 00000●000000 Evaluation 0 Summary 00

Executability

Where are the leaves of T ?

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 21 / 30

Theoretical Background 00000 What are we looking for? 00 Compactifying Decomposition Trees 00000 **SAT Encoding** 00000●000000 Evaluation 0 Summary 00

Executability

Where are the leaves of T ?

A leaf of T could be any vertex of the PDT ...

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 21 / 30

Theoretical Background 00000 What are we looking for? 00 Compactifying Decomposition Trees 00000 **SAT Encoding** 00000●000000 Evaluation 0 Summary 00

Executability

Where are the leaves of T ?

A leaf of T could be any vertex of the PDT ...
“inherit” them towards the leaves!

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 21 / 30

Theoretical Background 00000 What are we looking for? 00 Compactifying Decomposition Trees 00000 **SAT Encoding** 00000●000000 Evaluation 0 Summary 00

Executability

Encoding PDTs

$$\mathcal{F} = \mathcal{F}(r) \wedge c_i^j \tag{6}$$

$$\mathcal{F}(v) = \mathbb{M}(\{t^v \mid t \in \alpha(v)\}) \wedge \mathbb{M}(\{m^v \mid M(\alpha(v) \cap C)\}) \wedge \text{selectedMethod}(v) \wedge \text{applyMethod}(v) \wedge \text{nonePresent}(v) \tag{5}$$

$$\text{selectedMethod}(v) = \left[\bigwedge_{m \in M(\alpha(v) \cap C)} (m^v \rightarrow t^v) \right] \wedge \left[\bigwedge_{t \in \alpha(v) \cap C} \left(t^v \rightarrow \bigvee_{m \in M(t)} m^v \right) \right] \tag{1\&2\&4}$$

$$\text{applyMethod}(v) = \bigwedge_{m=(t,tn) \in M(\alpha(v))} \left[m^v \rightarrow \left(\bigwedge_{i=1}^{|tn|} t_{tn,i}^{v_i} \wedge \bigwedge_{v_i \in E(v) \setminus \{v_1, \dots, v_{|tn|}\}} \bigwedge_{t_i \in \alpha(v)} \neg t_i^{v_i} \right) \right] \tag{3}$$

$$\text{nonePresent}(v) = \left(\bigwedge_{t \in \alpha(v)} \neg t^v \right) \rightarrow \left(\bigwedge_{v_i \in E(v)} \bigwedge_{t \in C \cup P} \neg t^{v_i} \right) \wedge \bigwedge_{t \in \alpha(v) \cap P} \left(t^v \rightarrow \bigwedge_{v_i \in E(v)} \bigwedge_{t \in C \cup P} \neg t^{v_i} \right) \tag{4\&2}$$

$\alpha(v)$ is the set of labels of each vertex of the PDT.
 $E(v)$ are the children of v in the PDT.
 For every method $m = (c, tn)$, let v_i be the child to which the task $t_{tn,i}$ is assigned.
 $\mathbb{M}(A)$ is any encoding of the at-most-one constraint over the set of decision variables A .

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 22 / 30

Theoretical Background 00000 What are we looking for? 00 Compactingifying Decomposition Trees 00000 SAT Encoding 000000000000 Evaluation 0 Summary 00

Executability

Encoding PDTs

$$\mathcal{F} = \mathcal{F}(r) \wedge c_f^i \quad (6)$$

$$\mathcal{F}(v) = \mathbb{M}(\{t^v \mid t \in \alpha(v)\}) \wedge \mathbb{M}(\{m^v \mid M(\alpha(v) \cap C)\}) \wedge \text{selectedMethod}(v) \wedge \text{applyMethod}(v) \wedge \text{nonePresent}(v) \wedge \text{inheritPrimitive}(v) \quad (5)$$

$$\text{selectedMethod}(v) = \left[\bigwedge_{m \in M(\alpha(v) \cap C)} (m^v \rightarrow t^v) \right] \wedge \left[\bigwedge_{t \in \alpha(v) \cap C} \left(t^v \rightarrow \bigvee_{m \in M(t)} m^v \right) \right] \quad (1\&2\&4)$$

$$\text{applyMethod}(v) = \bigwedge_{m=(t,tn) \in M(\alpha(v))} \left[m^v \rightarrow \left(\bigwedge_{i=1}^{|tn|} t_{tn,i}^{v_i} \wedge \bigwedge_{v_i \in E(v) \setminus \{v_1, \dots, v_{|tn|}\}} \bigwedge_{t_* \in \alpha(v)} \neg t_*^{v_i} \right) \right] \quad (3)$$

$$\text{nonePresent}(v) = \left(\bigwedge_{t \in \alpha(v)} \neg t^v \right) \rightarrow \left(\bigwedge_{v_i \in E(v)} \bigwedge_{t \in \text{CUP}} \neg t^{v_i} \right) \quad (4\&2)$$

$$\text{inheritPrimitive}(v) = \bigwedge_{p \in \alpha(v) \cap P} \left[p^v \rightarrow \left(p^{v_1} \wedge \bigwedge_{v_i \in E(v) \setminus \{v_1\}} \bigwedge_{k \in \alpha(v)} \neg k^{v_i} \right) \right]$$

$\alpha(v)$ is the set of labels of each vertex of the PDT.
 $E(v)$ are the children of v in the PDT.
 For every method $m = (c, tn)$, let v_i be the child to which the task $t_{tn,i}$ is assigned.
 $\mathbb{M}(A)$ is any encoding of the at-most-one constraint over the set of decision variables A .

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 22 / 30

Theoretical Background 00000 What are we looking for? 00 Compactingifying Decomposition Trees 00000 SAT Encoding 000000000000 Evaluation 0 Summary 00

Executability

- β assigns primitive tasks to some leaves of the PDT
- A solution is an executable linearization of these tasks

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 23 / 30

Theoretical Background 00000 What are we looking for? 00 Compactingifying Decomposition Trees 00000 SAT Encoding 000000000000 Evaluation 0 Summary 00

Executability

Executability

- β assigns primitive tasks to some leaves of the PDT
- A solution is an executable linearization of these tasks
- Linearization has to be compatible with the ordering represented by the SOG

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 23 / 30

Theoretical Background 00000 What are we looking for? 00 Compactingifying Decomposition Trees 00000 SAT Encoding 000000000000 Evaluation 0 Summary 00

Executability

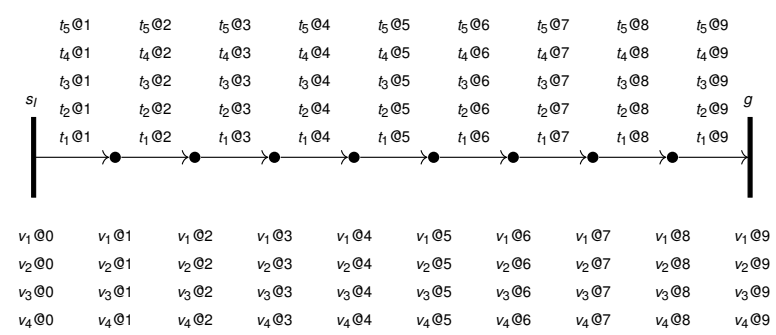
Executability

- β assigns primitive tasks to some leaves of the PDT
- A solution is an executable linearization of these tasks
- Linearization has to be compatible with the ordering represented by the SOG
- We represent a matching of the leaves to a sequence of timesteps and assert the correct order
- We can use any classical encoding of executability!

Chapter: Solving Hierarchical Planning Problems via SAT by Gregor Behnke Winter Term 2018/2019 23 / 30

Executability

Reminder: SAT Planning for Classical Problems – Decision Variables



Two types of decision variables!

- 1 t@i – Action t is executed at time i.
- 2 v@i – State variable v is true at time i.



Executability

Executability – Matching Leafs to Timesteps

- $\bar{\ell}i$ – the leaf ℓ is matched to timestep i
- a^ℓ – the leaf ℓ is active, i.e. a task is assigned to it

$$\mathcal{F}_{exe} = F_1 \wedge F_2 \wedge F_3 \wedge F_4 \wedge F_5 \wedge F_6$$

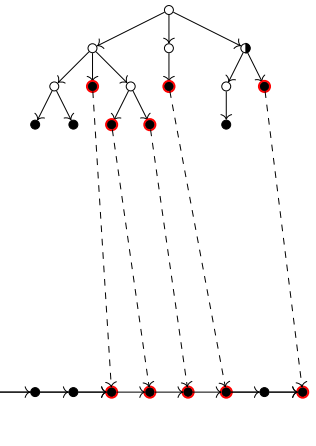
$$F_1 = \bigwedge_{i=1}^{|\mathcal{L}|} \mathbb{M}(\{\bar{\ell}i \mid \ell \in \mathcal{L}\}) \wedge \bigwedge_{\ell \in \mathcal{L}} \mathbb{M}(\{\bar{\ell}i \mid 1 \leq i \leq L\})$$

$$F_2 = \bigwedge_{\ell \in \mathcal{L}} \left[\left(\neg a^\ell \rightarrow \bigwedge_{p \in \alpha(\ell)} \neg p^\ell \right) \wedge \left(a^\ell \rightarrow \bigvee_{p \in \alpha(\ell)} p^\ell \right) \right]$$

$$F_3 = \bigwedge_{\ell \in \mathcal{L}} \left[\left(\neg a^\ell \rightarrow \bigwedge_{1 \leq i \leq |\mathcal{L}|} \neg \bar{\ell}i \right) \wedge \left(a^\ell \rightarrow \bigvee_{1 \leq i \leq |\mathcal{L}|} \bar{\ell}i \right) \right]$$

$$F_4 = \bigwedge_{\ell \in \mathcal{L}} \bigwedge_{t \in \alpha(\ell)} \bigwedge_{1 \leq i \leq |\mathcal{L}|} t^\ell \wedge \bar{\ell}i \rightarrow t@i$$

$$F_5 = \bigwedge_{1 \leq i \leq |\mathcal{L}|} \left[\left(\bigwedge_{\ell \in \mathcal{L}} \bar{\ell}i \right) \rightarrow \left(\bigwedge_{t \in \mathcal{O}} \neg t@i \right) \right]$$

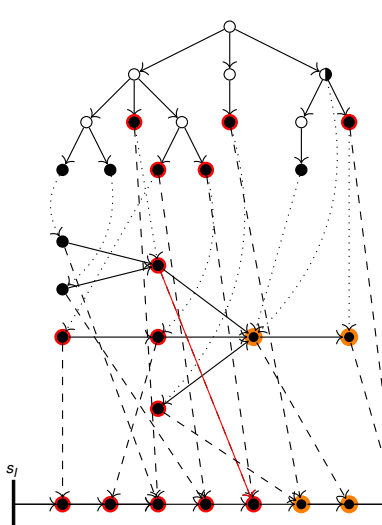


\mathcal{L} is the set of leaves of the PDT.



Executability

что делает? – Checking Order



- So far, the matching does not check the order imposed by the methods.
- Since SOG S is fixed: If leaf l is matched to time t , all successors of l must be matched to time after t , i.e. cannot be matched to times before t
- Using this property, we can reduce to $\mathcal{O}(n^3)$ clauses
- Often degenerates to $\mathcal{O}(n^2)$



Executability

Executability

- f_i^ℓ – matching the leaf ℓ to timestep i is forbidden (and implicitly also to any previous timestep)

$$F_6 = \bigwedge_{\ell \in \mathcal{L}} \bigwedge_{1 \leq i \leq |\mathcal{L}|} f_1(\ell, i) \wedge f_2(\ell, i) \wedge f_3(\ell, i) \wedge f_4(\ell, i)$$

$$f_1(\ell, i) = \text{if } i = 1 \text{ then true else } \bigwedge_{\ell' \in N_S^+(\ell)} \bar{\ell}'i \rightarrow f_{i-1}^{\ell'}$$

$$f_2(\ell, i) = \bigwedge_{\ell' \in N_S^+(\ell)} f_i^{\ell'} \rightarrow f_i^\ell$$

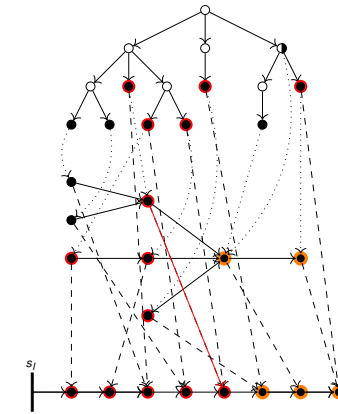
$$f_3(\ell, i) = \text{if } i = 1 \text{ then true else } f_i^\ell \rightarrow f_{i-1}^\ell$$

$$f_4(\ell, i) = f_i^\ell \rightarrow \neg \bar{\ell}i$$

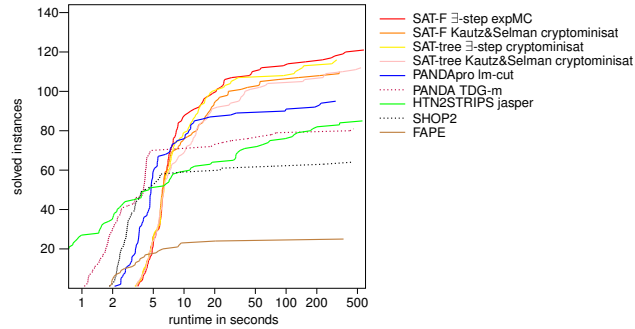
\mathcal{L} is the set of leaves of the PDT.

S is the Solution Order Graph.

$N_G^+(\ell)$ are the direct successors of vertex ℓ in the graph G .



Evaluation – Partially-Ordered Problems [Behnke, Hiller, Biundo, 2019]



	#instances	SAT-F \exists -step expMC	SAT-F Kautz&Selman cryptominisat	SAT-tree \exists -step cryptominisat	SAT-tree Kautz&Selman cryptominisat	PANDApro Im-cut	PANDA TDG-m	HTN2STRIPS jasper	SHOP2	FAPE
UM-TRANSLOG	22	22	22	22	22	22	22	22	22	22
SATELLITE	25	25	25	25	24	25	25	25	25	25
WOODWORKING	11	11	11	11	11	11	11	11	11	11
SMARTPHONE	7	7	7	7	7	7	7	7	7	7
PCP	17	12	12	12	12	12	12	12	12	12
ENTERTAINMENT	12	12	12	12	12	12	12	12	12	12
ROVER	20	10	11	9	8	5	6	4	4	4
TRANSPORT	30	22	20	20	15	14	15	17	22	20
Total	144	121	120	118	117	108	108	107	110	114



Even undecidable problems can be solved via a translation into SAT.

We have introduced

- Path Decomposition Trees (PDTs)
- Solution Order Graphs (SOGs)
- An encoding for PDTs and SOGs into propositional logic



References

Behnke,Höller,Biundo, 2018 totSAT – Totally-ordered hierarchical planning through SAT
 Behnke,Höller,Biundo, 2018 Tracking Branches in Trees – A Propositional Encoding for Solving Partially-Ordered HTN Planning Problems
 Behnke,Höller,Biundo, 2019 Bringing order to chaos – A compact representation of partial order in SAT-based HTN planning

