Game Playing AI

Guest Lecture in Structured Programming

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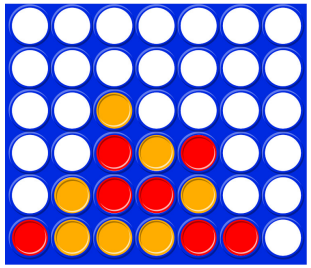
Outline for Today

- Motivation: Why Solving Games Automatically Anyways?
- What are Games? (A few Definitions)
- Solving Small Games
  - MiniMax
  - α/β Pruning
- Games with Chance
- Solving Large Games
- Game AI Success Story

Motivation

Why Bother? Why Solving Games Automatically?

- Game AIs for computer games (modern ones or board game adaptations).
- Purely for the sake of knowledge!
  E.g., can you always (force a) win in “Connect 4” when you start?



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- Game AIs for computer games (modern ones or board game adaptations).
- Purely for the sake of knowledge!
  E.g., can you always (force a) win in “Connect 4” when you start?
- Because many real-world problems can be regarded a game!
  The other player(s) in the game might be other agents or surroundings.
  - Economics! Cf. **game theory** (look up: *Nash Equilibrium* and *Prisoner's Dilemma*)
  - Robotics or Multi-Agent-Planning (though this is often cooperative, whereas we take a look at antagonistic games)

Games?

What are Games? Which Kinds Exist?

A **game** consists of a set of one or more **players**, a set of **moves** for the players, and a specification of **payoffs** (outcomes) for each combination of **strategies** (also called policy).

What kinds of restrictions can games have?

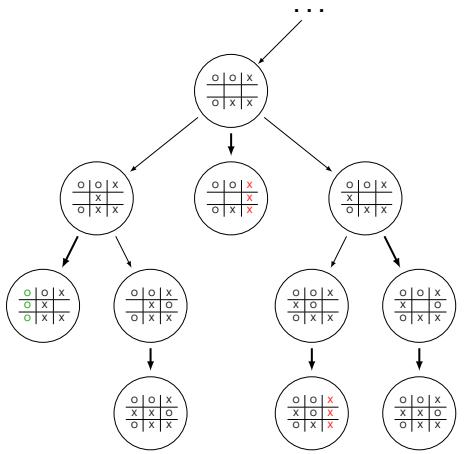
- Perfect information vs. imperfect information
- (One-player games vs.) Two-player games vs. multi-player games
- Zero-sum games vs. non-zero-sum games
- Games with chance (randomness) vs. games without chance



What's a Strategy?

A **strategy** defines a complete plan of action for a given player.

Given enough processing time an **optimal strategy** can be found for games of **perfect information** by enumerating paths of a **game tree**. However, in practice this can only be done for small games.



What are we Looking For?

What are we looking for?

- Game AI (strategy) vs. game theoretic outcome!
- Just because we have an AI that beats all humans, it doesn't mean the game is solved!

What's the game theoretic outcome?

- The outcome of the game assuming all players play *rational*.
- Rationality = optimization of expected reward.
- Outcome is known? $\rightarrow$ The respective game is "solved".



Solving Small Games

MiniMax: How to Solve Small Games?

Using search to solve a game:

- If the game tree is "sufficiently small" we can search in it to find and extract a strategy.
- But we still need to do that *efficiently*!

Consider two players, MAX and MIN. MAX tries to maximize his/her own score, and player MIN tries to minimize it.

We assume that the players are rational, taking turns, and that the game is zero-sum.

MiniMax: The MiniMax Algorithm

The MiniMax algorithm allows each player to compute their optimal move on a game tree of alternating MAX and MIN nodes.

The value of a node is the payoff for a game that is played optimally from that node until the end of the game.

max-value(s)
  if *state s is a leaf* **then**
    return payoff(s)
  $v := -\infty$
  forall *successor states s' of s* **do**
    $v := \max\{v, \text{min-value}(s')\}$
  return v

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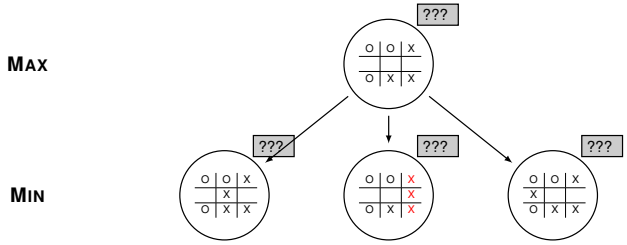

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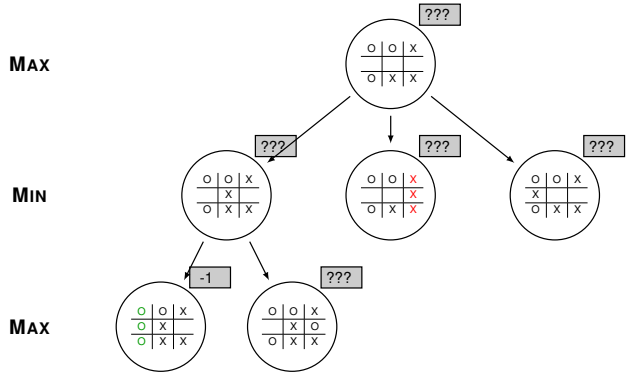

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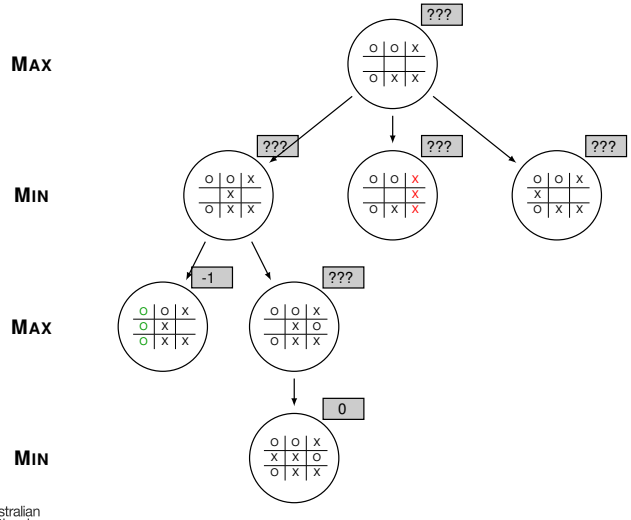

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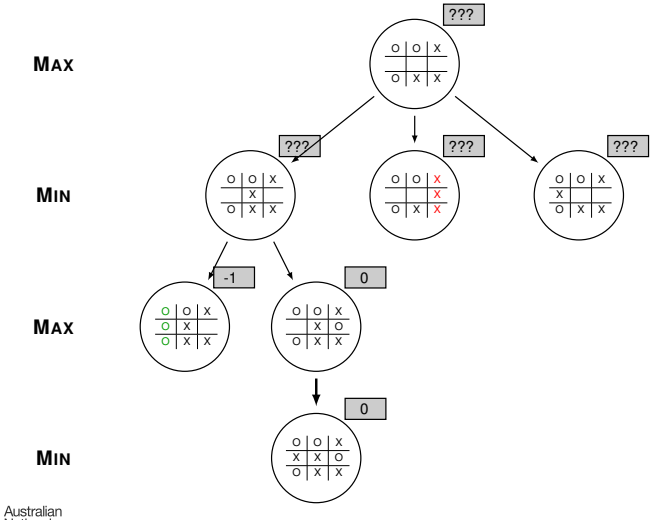

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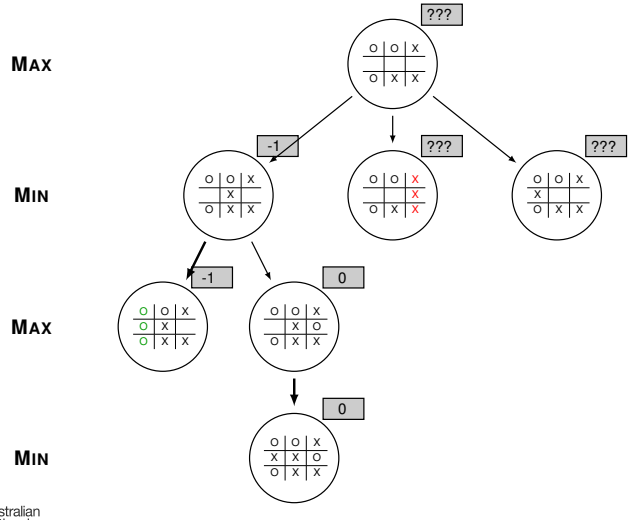

Motivation Games? Solving Small Games Games with Chance Solving Large Games Game AI Success Story

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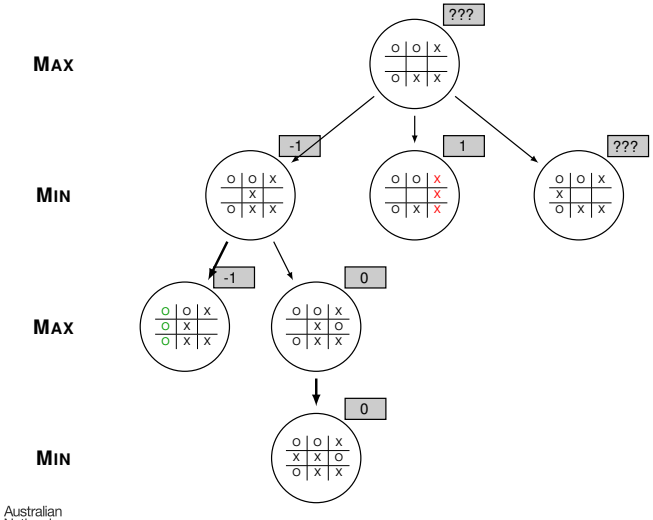

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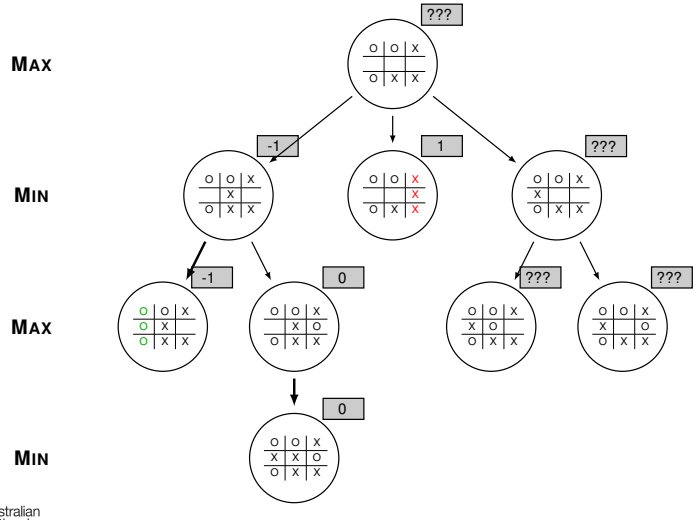

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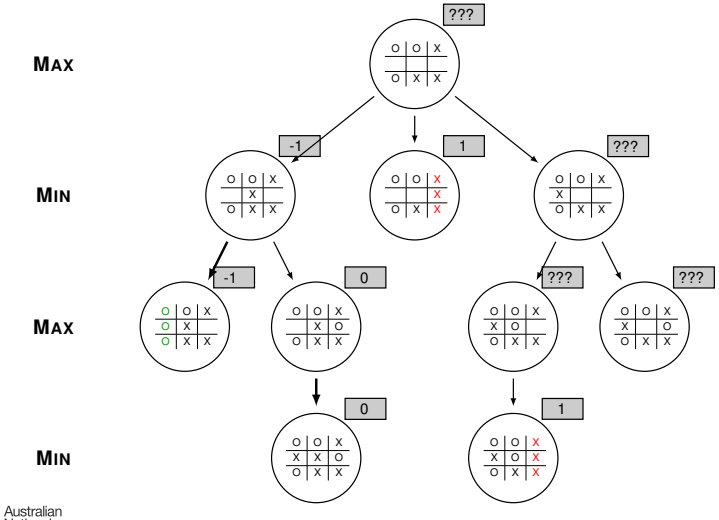

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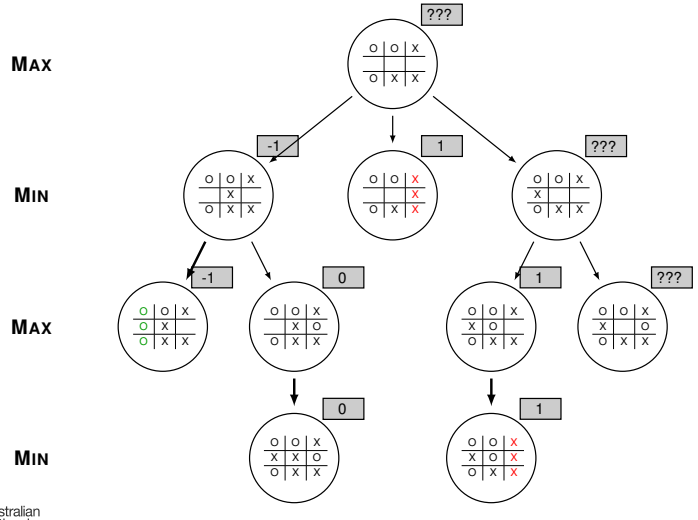

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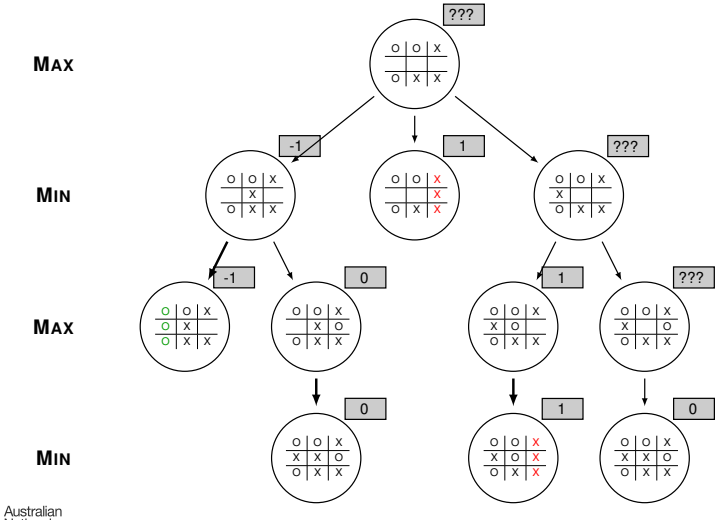

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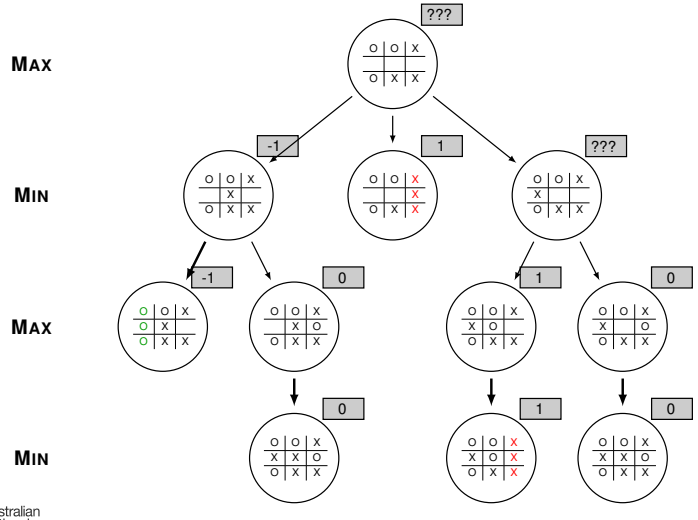

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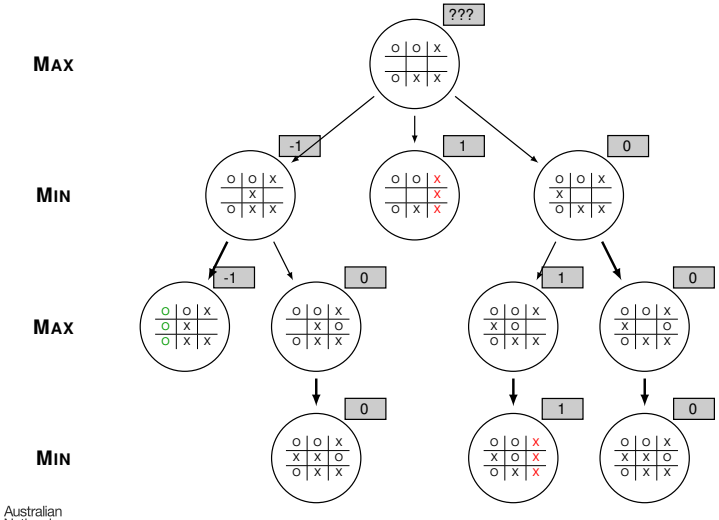

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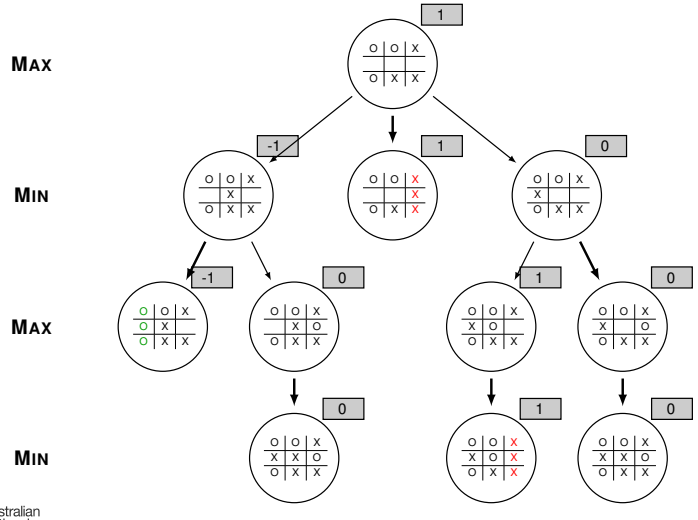

MiniMax: Space and Time Complexities

What is the runtime of MiniMax?

- Time: All nodes have to be visited! How many are there?
- Assume each game ends after d moves (tree depth). Each player has at most b moves (branching factor)

$\rightarrow$ Runtime is in $O(b^d)$ (exponential!)

What is the space requirement of MiniMax?

- We perform a depth-first search!
- So only the longest path needs to be stored.

$\rightarrow$ Space is in $O(b \cdot d)$ (linear)

α/β Pruning: Can we do better?

- MiniMax suffers from the problem that the number of game states it has to examine is *always* exponential in the number of moves.
- α/β pruning is a method for reducing the number of nodes that need to be evaluated by only considering nodes that may be reached in game play.
- Alpha-beta pruning places bounds on the values appearing anywhere along a path:
  - α is the best (highest) value found so far for MAX
  - β is the best (lowest) value found so far for MIN

  α and β propagate down the game tree.
  v propagates up the game tree.

α/β Pruning: The MiniMax Algorithm Extended By α/β Pruning

Keep in mind:

- α is the best value found so far for MAX, initialize with $-\infty$.
- β is the best value found so far for MIN, initialize with ∞.

max-value(s, α, β)
  if *state s is a leaf* **then**
    return payoff(s)
  $v := -\infty$
  forall *successor states* s' *of* s **do**
    $v := \max\{v, \text{min-value}(s', \alpha, \beta)\}$
    if $v \geq \beta$ **then**
      return v
    $\alpha := \max\{\alpha, v\}$
  return v

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  $v := \infty$
  forall *successor states* s' *of* s **do**
    $v := \min\{v, \text{max-value}(s', \alpha, \beta)\}$
    if $v \leq \alpha$ **then**
      return v
    $\beta := \min\{\beta, v\}$
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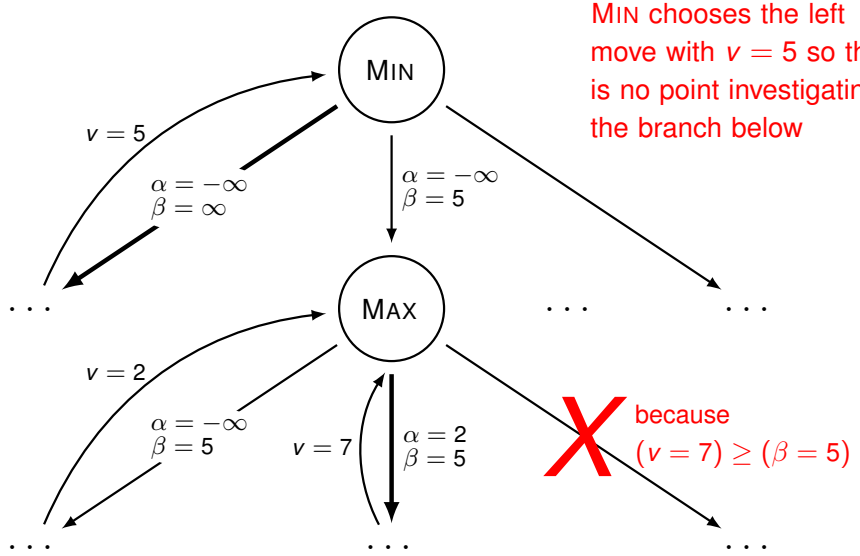
α/β Pruning: Idea Behind Pruning: When and Why?



MIN chooses the left move with $v = 5$ so there is no point investigating the branch below

α/β Pruning: Example: Tic Tac Toe

Start with $\alpha = -1$ (rather than $-\infty$) and $\beta = 1$ (rather than ∞)

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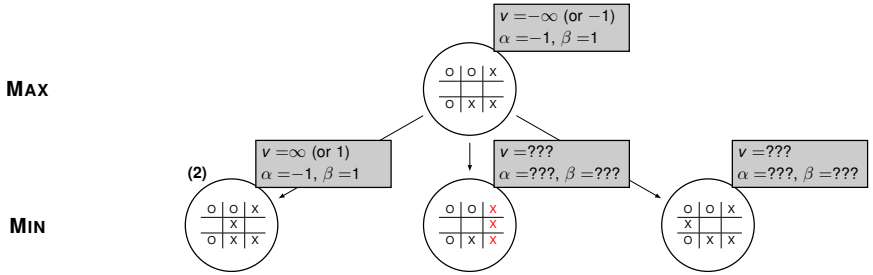

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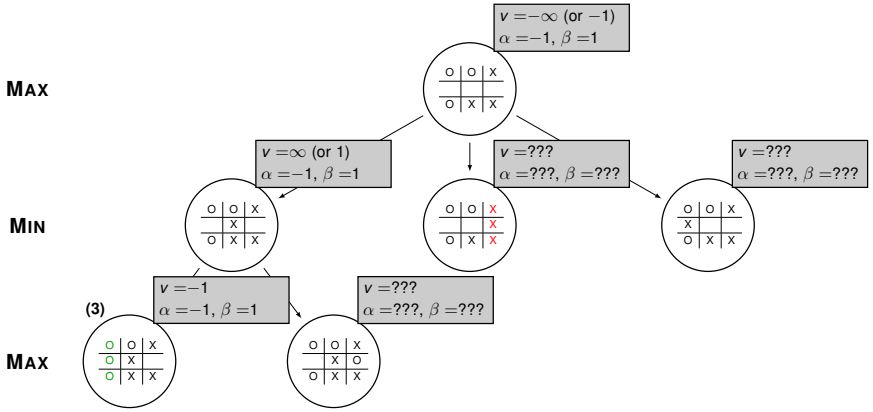

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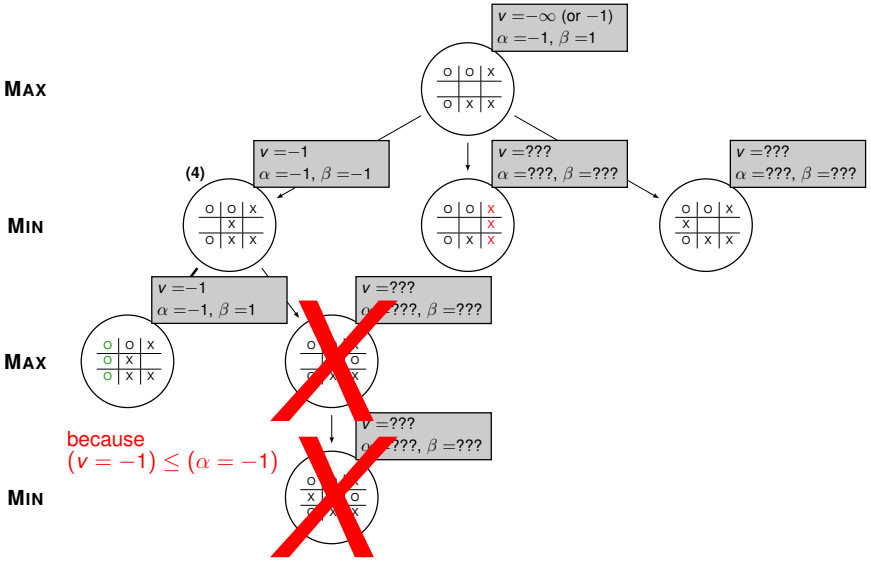

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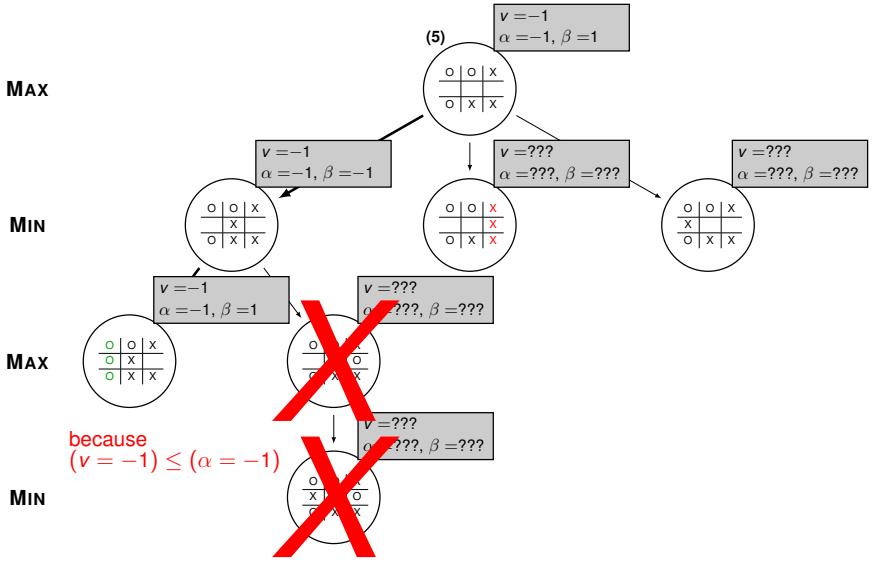

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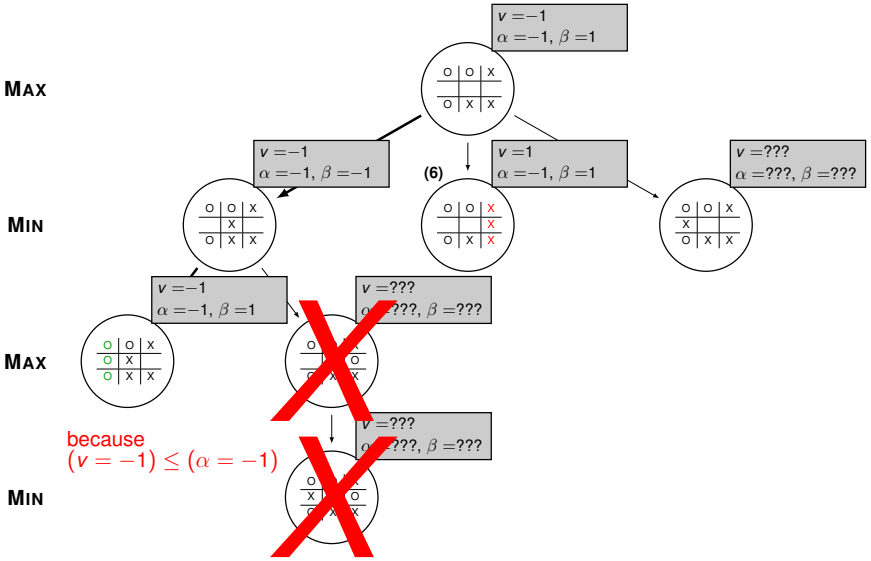

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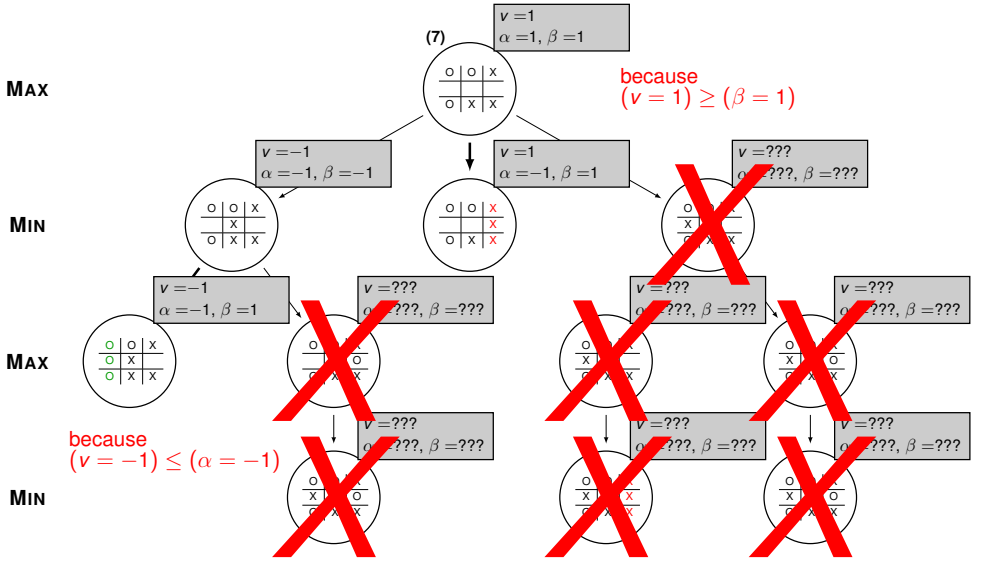
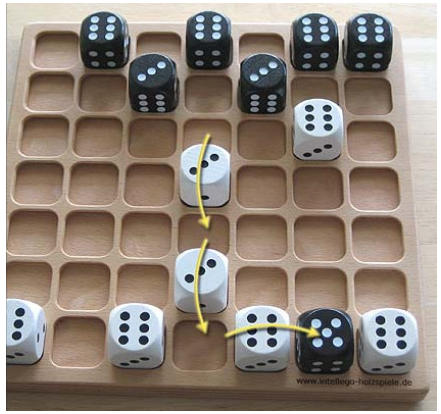


α/β Pruning: Space and Time Complexities

What is the runtime (and space requirements) of α/β pruning?

- In the worst case: identical to MiniMax! If nothing can be pruned.
- On average: Complexities omitted. (Due to lack of time.)
- This can happen depending on the order in which edges are traversed/payoffs are discovered.
- In practice, it is very unlikely that no pruning occurs, so *always* choose α/β pruning over MiniMax!

α/β Pruning: Assignment: AI for Cublino



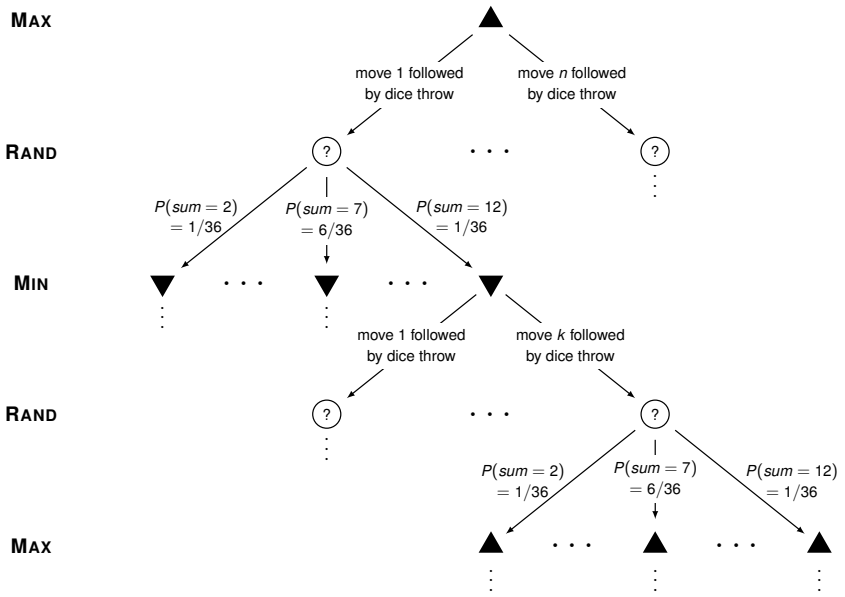
- Perfect information, deterministic, zero-sum
- Branching factor? (Much higher than in, e.g., Connect 4)
- Game horizon? (I guess a bit higher than in Connect 4)

Games with Chance

How to Deal with Randomness?

- A random decision can be regarded as the move of yet another player!
- Certainly that's not another MAX player! I.e., the "environment" (the random decision) will not always play in our favor!
- But what is it, then?
  - Another MIN player? (Too pessimistic...)
  - If we want to play *rational*, we maximize the expectation!
    $value(s) = \sum_{\text{successor states } s' \text{ of } s} P(s') \cdot value(s')$

Illustration For a 2-Player Game With Throwing Two Dice, Counting Their Sum



Solving Large Games

The "Size" of Games

When is using MiniMax and α/β Pruning still feasible?

- Recall that the complexity of MiniMax (and α/β!) is exponential! I.e., in $O(b^d)$, with
  - b, the branching factor (available moves per state)
  - d, the depth (number of moves until game ends)
- For some games that is simply too large!
- So, let's take a look at some examples...

The "Size" of Games: Tic Tac Toe

Examples for (estimated) number of reachable (game) states:

(Source: `https://en.wikipedia.org/wiki/Game_complexity`)

- Rough maximum: $3^9 = 19,683$ (including invalid states)
- Actual maximum: $5,478$
- Maximum after duplicating symmetries: 765
- There are still $26,830$ possible *games*!
  (For those states with eliminated duplicates.)
  What's a "game"?
  A path in the MiniMax tree!

The "Size" of Games: Connect 4

Examples for (estimated) number of reachable (game) states:

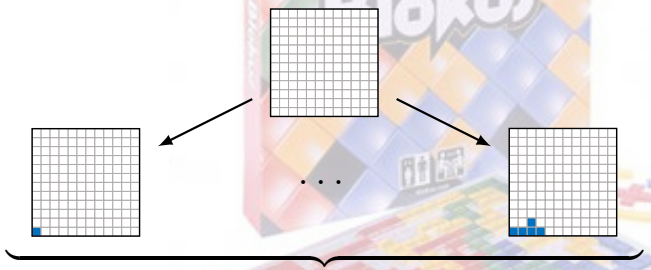
(Source: `https://en.wikipedia.org/wiki/Connect_Four`)

- Rough maximum: $3^{7 \cdot 6} < 1.1 \cdot 10^{20}$ (including invalid states)
- Actual maximum: $4,531,985,219,092 \approx 4.5 \cdot 10^{12}$ (still including symmetries)
- First solved, independently, by James Dow Allen (October 1, 1988), and Victor Allis (October 16, 1988).
- Note that today it can also be solved using α/β pruning!

The "Size" of Games: Blokus

Examples for (estimated) number of reachable (game) states:

(Source: by Stephen Gould, previous year(s))



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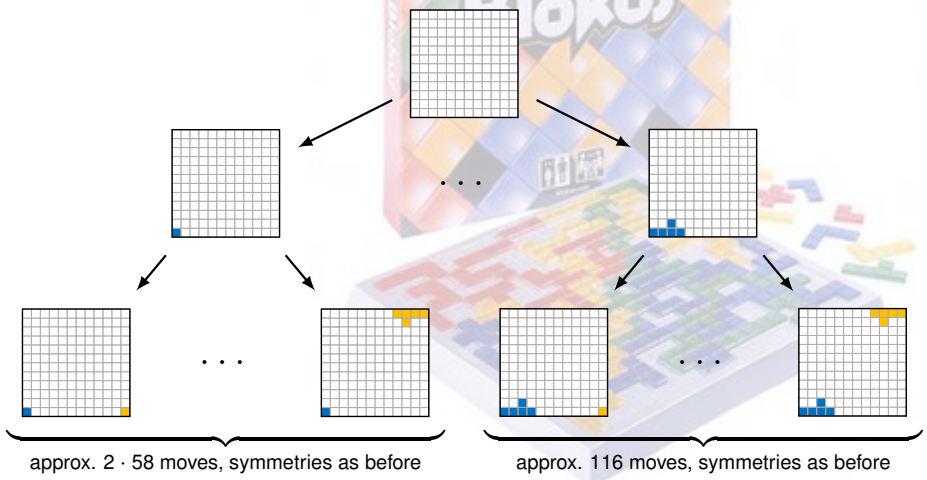

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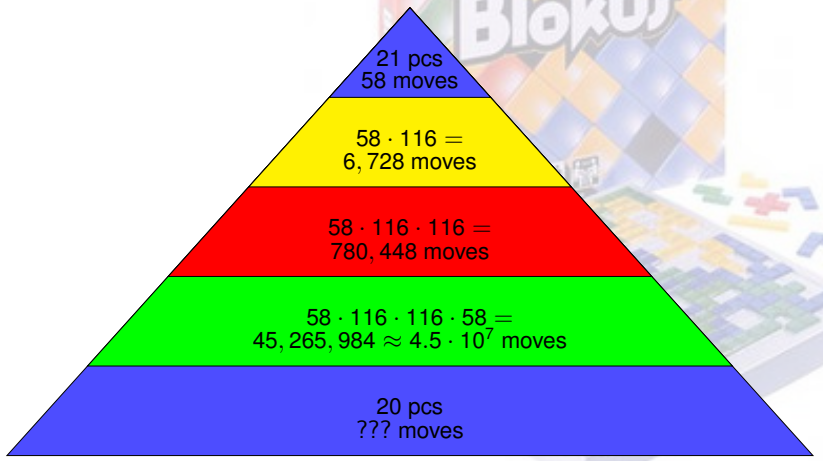
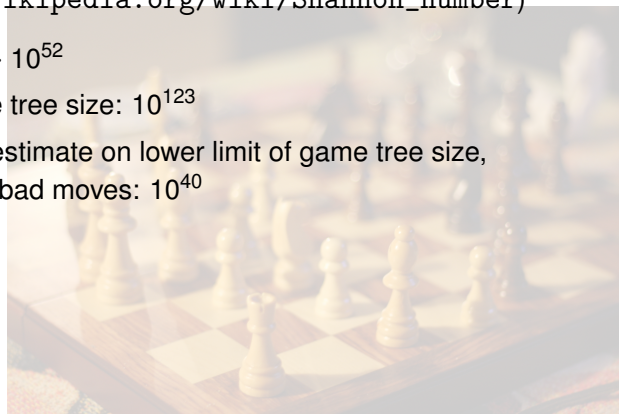


The "Size" of Games: Chess

Examples for (estimated) number of reachable (game) states:

(Source: `https://en.wikipedia.org/wiki/Shannon_number`)

- Some maximum: $5 \cdot 10^{52}$
- Lower limit on game tree size: 10^{123}
- More conservative estimate on lower limit of game tree size, eliminating obvious bad moves: 10^{40}

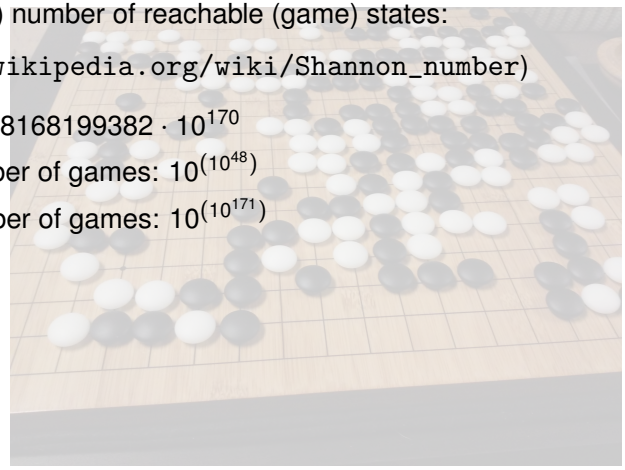


The "Size" of Games: Go

Examples for (estimated) number of reachable (game) states:

(Source: `https://en.wikipedia.org/wiki/Shannon_number`)

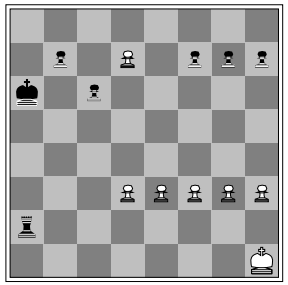
- Legal positions: $2.08168199382 \cdot 10^{170}$
- Lower limit on number of games: $10^{(10^{48})}$
- Upper limit on number of games: $10^{(10^{171})}$



How to deal with large games?

So, what to do for (too) large games?

- Don't compute the entire game tree!
- Stop at certain nodes and *estimate* their payoff! But how?
  - hand-crafted heuristics



Black to move

Title: *Artificial Intelligence: A Modern Approach (3rd Ed.)*
Authors: *Stuart Russel* and *Peter Norvig*
URL: `https://aima.cs.berkeley.edu/`

Estimate per piece:

- pawn: 1 pt
- knight/bishop: 3 pts
- rook: 5 pts
- queen: 9 pts

Estimate: Black: 7 pts versus White: 6 pts
→ Black leading! (Only *very* slightly.)

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- Don't compute the entire game tree!
- Stop at certain nodes and *estimate* their payoff! But how?
  - hand-crafted heuristics
  - learned heuristics

Machine learning techniques are often used to find a good static evaluation function based on a linear combination of features:

$$\hat{v}(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

Note the similarity to chess!

- $w_1 = 1$, $f_1(s) =$ number of pawns in s
- $w_2 = 3$, $f_2(s) =$ number of knights/bishops in s
- ...

How to deal with large games?

So, what to do for (too) large games?

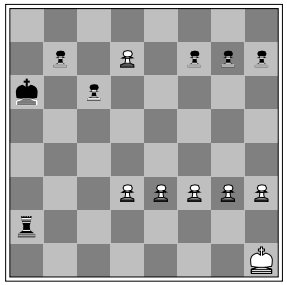
- Don't compute the entire game tree!
- Stop at certain nodes and *estimate* their payoff! But how?
  - hand-crafted heuristics
  - learned heuristics
  - simulate a game, use the outcome as estimate

  Monte-Carlo Tree Search is a well-known algorithm exploiting this idea. It works in four phases:
  - *Selection* (select a non-terminal leaf based on current strategy)
  - *Expansion* (expand the selected node)
  - *Simulation* (play a random game to the end)
  - *Backpropagation* (use the outcome to update strategy)

  Interested? See, e.g.,
  `https://www.youtube.com/watch?v=UXW2yZndl7U`
  (15:30, lecture by Dr. John Levine from Univ. of Strathclyde)

When to use heuristics?

- In standard MiniMax or *alpha*/*beta* pruning, we make a **terminal test** to obtain the payoff, or continue expanding. With heuristics, we instead make a **cut-off test** to check whether we should stop expansion and *estimate* the payoff of the current node.
- What about using a fixed depth as cut-off test? → Suffers from the **horizon problem**:



Black to move

Title: *Artificial Intelligence: A Modern Approach (3rd Ed.)*
Authors: *Stuart Russel* and *Peter Norvig*
URL: `https://aima.cs.berkeley.edu/`

White can promote a pawn into a queen on its next move! So the cut-off test should be negative in this state.

Game AI Success Story

Mile Stones in AI Game Playing

1959 Arthur Samuel develops Checkers playing program

1997 IBM's Deep Blue chess machine beats Garry Kasparov

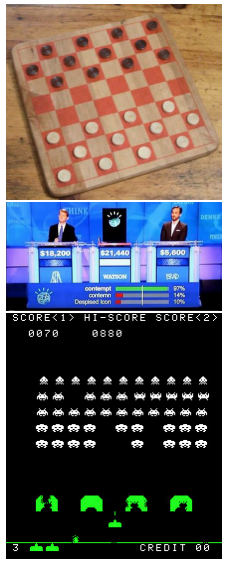
2007 Checkers solved by University of Alberta

2011 IBM's Watson wins Jeopardy! requiring natural language understanding

2015 Deep reinforcement learning algorithms learn to play Atari arcade games from scratch

2016 Google DeepMind's AlphaGo beats Lee Sedol, Korea

2017 AlphaZero learns Go, Chess, and Shogi from scratch (and beats AlphaGo)



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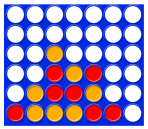

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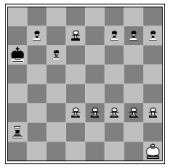

By Stuart Russel and Peter Norvig
from their book *Artificial Intelligence: A Modern Approach (3rd Ed.)*
Availble freely for teaching on `https://aima.cs.berkeley.edu/`

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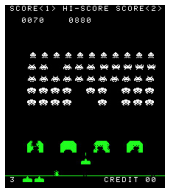

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