

Logic (PHIL 2080, COMP 2620, COMP 6262)
Chapter: Cheat Sheet for all Rules
(Weeks 1 to 6)

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Propositional Logic: Conjunction

Conjunction Elimination:

$$\frac{X \vdash A \wedge B}{X \vdash A} \wedge E$$

$$\frac{X \vdash A \wedge B}{X \vdash B} \wedge E$$

Conjunction Introduction:

$$\frac{X \vdash A \quad Y \vdash B}{X, Y \vdash A \wedge B} \wedge I$$



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Propositional Logic: Implication

Implication Elimination:

$$\frac{X \vdash A \rightarrow B \quad Y \vdash A}{X, Y \vdash B} \rightarrow E$$

Implication Introduction:

$$\frac{X, A \vdash B}{X \vdash A \rightarrow B} \rightarrow I$$



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Propositional Logic: Negation

Double-Negation Elimination and Introduction:

$$\frac{X \vdash \neg\neg A}{X \vdash A} \neg\neg E \qquad \frac{X \vdash A}{X \vdash \neg\neg A} \neg\neg I$$

(Single) Negation Elimination and Introduction:

$$\frac{X \vdash A \quad Y \vdash \neg A}{X, Y \vdash \perp} \neg E \qquad \frac{X, A \vdash \perp}{X \vdash \neg A} \neg I$$

Reductio ad Absurdum (RAA):

$$\frac{X, B \vdash A \quad Y, B \vdash \neg A}{X, Y \vdash \neg B} RAA$$

Propositional Logic: Disjunction

Disjunction Introduction and Elimination:

$$\frac{X \vdash A}{X \vdash A \vee B} \vee I \qquad \frac{X \vdash B}{X \vdash A \vee B} \vee I$$

$$\frac{X \vdash A \vee B \quad Y, A \vdash C \quad Z, B \vdash C}{X, Y, Z \vdash C} \vee E$$

Predicate Logic: Existential Quantifier

Existential Introduction Rule:

$$\frac{X \vdash A_x^t}{X \vdash \exists x A} \exists I \quad \text{Provided } t \text{ is not bound in } A_x^t$$

Existential Elimination Rule:

$$\frac{X \vdash \exists x A_x^t \quad Y, A \vdash B}{X, Y \vdash B} \exists E \quad \text{Provided } t \text{ does not occur in } B \text{ or any formula in } Y$$

Predicate Logic: Universal Quantifier

Universal Introduction Rule:

$$\frac{X \vdash A}{X \vdash \forall x A_x^v} \forall I \quad \text{Provided } v \text{ does not occur in } X$$

Universal Elimination Rule:

$$\frac{X \vdash \forall x A}{X \vdash A_x^t} \forall E \quad \text{Provided } t \text{ is not bound in } A_x^t$$

Semantic Tableau

Propositional Logic: Conjunction, Disjunction, and Negation Elimination

And Elimination:

$$\frac{T: A \wedge B}{T: A, T: B}$$

$$\frac{F: A \wedge B}{F: A \mid F: B}$$

Or Elimination:

$$\frac{T: A \vee B}{T: A \mid T: B}$$

$$\frac{F: A \vee B}{F: A, F: B}$$

Negation Elimination:

$$\frac{T: \neg A}{F: A}$$

$$\frac{F: \neg A}{T: A}$$

Implication Elimination:

$$\frac{T: A \rightarrow B}{F: A \mid T: B}$$

$$\frac{F: A \rightarrow B}{T: A, F: B}$$

Predicate Logic: Existential and Universal Quantifiers

$$\frac{F: \exists x Fx}{F: Fa, F: Fb, \dots}$$

for all a, b, ... in the branch – present and future!

$$\frac{T: \exists x Fx}{T: Fa}$$

if a is new to the branch

$$\frac{T: \forall x Fx}{T: Fa, T: Fb, \dots}$$

for all a, b, ... in the branch – present and future!

$$\frac{F: \forall x Fx}{F: Fa}$$

if a is new to the branch

≡

≡

≡

≡

$$\frac{X, F: \exists x A}{X, F: \exists x A, F: A_x^a}$$

for a in X or A

$$\frac{X, T: \exists x A}{X, T: A_x^a}$$

for a not in X or A

$$\frac{X, T: \forall x A}{X, T: \forall x A, T: A_x^a}$$

for a in X or A

$$\frac{X, F: \forall x A}{X, F: A_x^a}$$

for a not in X or A

Predicate Logic: Existential and Universal Quantifier for Invalid Sequents

$$\frac{\text{inv} \quad T: \exists x Fx}{T: Fa \mid T: Fb \mid \dots \mid T: Fn}$$

for all a, b, ... in the branch or n new to the branch

$$\frac{\text{inv} \quad F: \forall x Fx}{F: Fa \mid F: Fb \mid \dots \mid F: Fn}$$

for all a, b, ... in the branch or n new to the branch

≡

≡

$$\frac{\text{inv} \quad X, T: \exists x A}{X, T: A_x^a \mid X, T: A_x^b \mid \dots \mid X, T: A_x^n}$$

for any/all a, b, ... in X or A, or n not in X or A

$$\frac{\text{inv} \quad X, F: \forall x A}{X, F: A_x^a \mid X, F: A_x^b \mid \dots \mid X, F: A_x^n}$$

for any/all a, b, ... in X or A, or n not in X or A