COMP3630 / COMP6363

## week 12: Automated (HTN) Planning

(A subdiscipline of Artificial Intelligence)

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The Australian National University

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- Introduction to Hierarchical Planning
- Complexity Studies
- Expressivity Studies (also for Classical Planning)

## Terminology and some Background

- HTN Planning is short for Hierarchical Task Network Planning.
- It's an extension of classical planning where:
  - We don't plan for some goal but want to refine some initial tasks.
  - We also can't insert actions in every state, but need to adhere certain rules.
- Historical remarks:
  - Whereas first versions date back to the 70s, the first decent formalization comes from the early 90s.
  - Some central idea was to introduce expert knowledge: What do we need to do to achieve a certain task?
- Why defining/solving a hierarchical problem?
  - As above: In many real-world applications, knowledge is given in form of control rules: we know the steps required to perform some task.
  - More control on the generated plans, since all the "rules" need to be obeyed. We can exclude (more) undesired plans!
  - Plans can be presented more abstract by relying on task hierarchies.
  - We can solve/express more complex problems! (Spoiler)

# Example: Do-It-Yourself (DIY) Assistant, The Task



The material:

- Boards (need to be cut first)
- Electrical devices like drills and saws
- Attachments like drill bits and materials like nails

Further reading: Pascal Bercher et al. "Do It Yourself, but Not Alone: Companion-Technology for Home Improvement – Bringing a Planning-Based Interactive DIY Assistant to Life." Künstliche Intelligenz – Special Issue on NLP and Semantics, 35: 367–375. 2021.

week 12: Automated (HTN) Planning

## Example: Do-It-Yourself (DIY) Assistant, User Interface



# Example: Do-It-Yourself (DIY) Assistant, Task Hierarchy











$$\mathcal{P} = (V, P, \delta, C, M, s_l, c_l, g)$$

- V a set of state variables
- P a set of primitive task names
- $\delta: \mathcal{P} 
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- C a set of compound task names



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  - $M \subseteq C \times 2^{TN}$  the methods
- A solution task network tn must:
  - be a refinement of  $c_l$ ,
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- $c_I \in C$  the initial task
- $M \subseteq C \times 2^{TN}$  the methods
- $s_I \in 2^V$  the initial state
- $g \subseteq V$  the (optional) goal description
- A solution task network *tn* must:
  - be a refinement of  $c_l$ ,
  - only contain primitive tasks, and
  - have an executable linearization that makes the goals in g true.

## HTN Planning, Problem Definition

- Task network:  $tn = (T, \prec, \alpha)$  consists of:
  - T, a possibly empty set of task identifier symbols (IDs)
  - $\prec \subseteq T \times T$ , a partial order on the task IDs,
  - $\alpha: T \to P \dot{\cup} C$ , the task mapping function
- Executability of primitive tasks is defined as in classical planning. (But since we deal with task networks we also demand that the respective (primitive) task network possesses an executable linearization that makes the goals true.)
- A decomposition method  $m \in M$  is a tuple  $m = (c, tn_m)$  with a compound task cand task network  $tn_m = (T_m, \prec_m, \alpha_m)$
- Let  $tn = (T, \prec, \alpha)$  be a task network,  $t \in T$  a task identifier, and  $\alpha(t) = c$  a compound task to be decomposed by  $m = (c, tn_m)$ . We assume  $T \cap T_m = \emptyset$ . Then, the application of m to tn results into the task network  $tn' = ((T \setminus \{t\}) \cup T_m, \prec \cup \prec_m \cup \prec_X, \alpha \cup \alpha_m)|_{(T \setminus \{t\}) \cup T_m}$  with:

$$\prec_X := \{ (t', t'') \mid (t', t) \in \prec, t'' \in T_m \} \cup \\ \{ (t'', t') \mid (t, t') \in \prec, t'' \in T_m \}$$

where  $(X_1, \ldots, X_n)|_Y$  restricts the sets  $X_i$  to elements in Y.

# HTN Planning, Problem Definition (Solution Criteria)

A task network *tn* is a solution if and only if:

- There is a sequence of decomposition methods  $\overline{m}$  that transforms  $c_l$  into tn,
- tn contains only primitive tasks, and
- the (still partially ordered) task network tn admits an executable linearization  $\overline{t}$  of its tasks leading to some state  $s \supseteq g$ .



## HTN Planning is in RE

To prove RE membership, we give a partial decision procedure.

There are (at least) two:

- Systematically generate all refinements (e.g., via "progression search"). Accept if we found an executable one.
- Systematically generate all action sequences and verify whether they are executable and can be generated by the task hierarchy (via "plan verification"). Accept if we found an executable one that can be generated by the hierarchy.

Details omitted.

## HTN Planning is not in R

We reduce from the (undecidable) grammar intersection problem. (Chap.9, slide 31)

More specifically, deploy HTN planning to solve the following problem: Given context-free grammars G and G', is  $L(G) \cap L(G') = \emptyset$ ?

Decision procedure:

- $\bullet\,$  Construct an HTN planning problem  ${\cal P}$  that has a solution if and only if the correct answer is yes.
- Translate the production rules to decomposition methods in a way that only words in both L(G) and L(G') can be produced.
- Any solution *tn* contains only one executable linearization. Each such linearization  $\omega$  contains some  $\omega'$  twice, with  $\omega' \in L(G)$  and  $\omega' \in L(G')$ .

We show the encoding using an example. (And skip the proof that it's a reduction.) Proof by Erol et al., 1994.

## HTN Planning is not in R, cont'd



Pascal Bercher

#### Decidable Subcases

We only list some special cases that make HTN planning decidable.

- Acyclicity of Tasks. (Finitely many plans.)
- Total Order. (Among all the tasks.)
- Delete Relaxation.
- Regularity. (Only the last task in each method can be compound.)
- Tail-recursivity. (Generalization of Regularity.)
- Task insertion. (If we can also insert tasks anywhere.)
- Many more (possibly).

**Recap:** A language *L* is a set of strings over symbols.

- Let  $\mathcal{P}$  be a (classical) planning problem and  $sol(\mathcal{P})$  its set of solutions. If we interpret any action as a symbol, then  $sol(\mathcal{P})$  is a language!
- Recall that in HTN planning we had  $\delta: P \to (2^V)^3$ , so every action had a unique name. We thus assume this for classical planning as well. (So solutions are sequences of task names.)
- So we can define:
  - $L(\mathcal{P}) = sol(\mathcal{P})$  if  $\mathcal{P}$  is a classical problem.
  - $L(\mathcal{P}) = \{\bar{p} : tn \in sol(\mathcal{P}) \text{ and } \bar{p} \text{ is an executable linearization of } tn \text{ that makes } g \text{ true.} \}$
- We can now compare planning problems (and their special cases) with regard to the Chomsky Hierarchy (i.e., the standard language classes, like regular and context-free languages).

# HTN problems, defined differently

Let  $\ensuremath{\mathcal{P}}$  be a hierarchical planning problem.

- Let  $L_H(\mathcal{P}) = \{ \bar{p} : \bar{p} \text{ is the set of solutions to } \mathcal{P} \text{ when disregarding executability. } \}$
- Let  $L_C(\mathcal{P}) = \{\bar{p} : \bar{p} \text{ is the set of solutions to } \mathcal{P} \text{ the induced classical problem while disregarding the initial task network. }$
- Thus,  $L_H$  just looks at the 'words' produced by the hierarchy, whereas  $L_C$  just looks at the executable words that produce the goal.
- Now,  $L(\mathcal{P}) = L_H(\mathcal{P}) \cap L_C(\mathcal{P}).$

This obseration/proposition gives a new/simplified view on HTN planning:

#### HTN planning extends classical planning by adding a grammar to filter solutions.

## Classes of Planning Problems

We can define the following Language classes:

- Let  $\mathcal{HTN} = \{L(\mathcal{P}) : \mathcal{P} \text{ is an HTN planning problem.}\}$
- Let  $CLASSIC = \{L(P) : P \text{ is a classical planning problem.}\}$
- We can do the same for any restriction on planning problems:
  - $TOHTN = \{L(P) : P \text{ is a total-order HTN planning problem.}\}$
  - and for any other restriction!

#### Expressivity of Classical Problems

#### **Theorem:** $CLASSIC \subsetneq REG$

#### **Proof:**

- We first show  $CLASSIC \subseteq REG$ .
  - For this, notice that each planning problem encodes an (exponentially larger) DFA. Thus, given a classical planning problem, we can create its DFA. Each node is a state, each edge is an action.
  - We know that each DFA is regular, thus showing the claim.
- We now show  $CLASSIC \subsetneq REG$ .
  - We prove that  $\{aa\} \in REG$  is not the language of any classical problem  $\mathcal{P}$ ,  $L(\mathcal{P}) \neq \{aa\}$  for all  $\mathcal{P}$ .
  - Assume aa ∈ L(P) for some classical problem P. Then, we can show that aaa ∈ L(P). (We skip the proof here, just write down the sets to show that pre(a) must be contained in the state resulting from aa – because a was applicable in the state after executing a in s<sub>I</sub>.)

### Expressivity of HTN Problems

#### **Theorem:** TOHTN = CF

#### **Proof:**

- We first show  $\mathcal{TOHTN} \supseteq CF$ .
  - Let *G* be a CF grammar. Use rules as methods, compound task names as terminal symbols, and primitive task names as terminal symbols.
  - For each terminal symbol define a no-operation (i.e., empty preconditions and effects). Set  $g = \emptyset$ .
- Now we show  $TOHTN \subseteq CF$ .
  - We know that  $L(\mathcal{P}) = L_H(\mathcal{P}) \cap L_C(\mathcal{P})$  for all HTN problems  $\mathcal{P}$ .
  - We know that  $L_H(\mathcal{P})$  is context-free and that  $L_C(\mathcal{P})$  is regular.
  - It is known that the intersection of a context-free and regular language is context-free.

#### Conclusion

Some final remarks.

- On't forget that:
  - We have almost 10 (internationally known) AI Planning experts at the ANU. (In case you want to do a PhD or research project.)
  - Many (most?) in the Foundations group (might) have theory-heavy research projects to offer – assuming they have any project at all! (They/we might not have any at the moment, or no capacities left.)
- Please take part in SELT. (No matter whether you liked it or not.)
- I hope you enjoyed the course!
- Good luck in the exam! (And your other exams.)

Thank you for taking this course!