COMP3630 / COMP6363

week 12: **Automated (Classical) Planning**

(A subdiscipline of Artificial Intelligence)

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The Australian National University

Semester 1, 2023

Content of this Chapter

- Introduction to Classical Planning
- Complexity Studies

Disclaimer

Why do we have this week's content?

- I wanted to provide additional examples to strengthen your current understanding rather than including additional content. Compared to $\leq$ 2022 you will miss out on:
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  - To find PhD students! The ANU has at least 8 planning experts, and we are all internationally connected (in case you want to do research Overseas). But note that ANU's Foundations Cluster has just as much staff with theory-heavy topics!

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- An initial world description (start state)
- A desired world description (end state)
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- Any additional constraints on solution plans?

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- Is there time involved? we: No
- Any additional constraints on solution plans? we: No

Classical Planning is the simplest form of planning!

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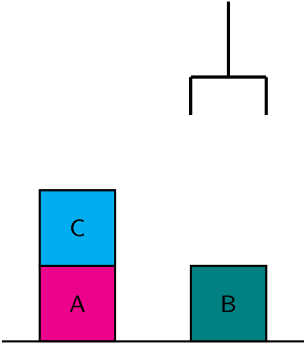
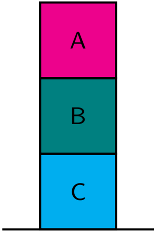
  Well... Yes for HTN planning!

Classical Planning is the simplest form of planning! But HTN Planning is more complex.

Artificial Toy Problems, e.g., Blocksworld



Start Configuration



Desired Configuration

- Standard Planning Benchmark in the International Planning Competition

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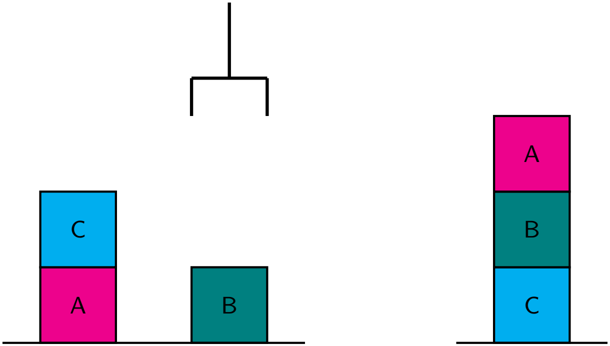

Start Configuration | Desired Configuration

- Standard Planning Benchmark in the International Planning Competition
- . . . and every planning lecture! (Like this and the one below.)
- Here (`https://www.youtube.com/watch?v=pfNb0IAkbcQ&t=308s`) you find a 90 minute hands-on lecture by me on modeling Blocksword using planning. (I.e., you will actually model it during the lecture and use an online planner to solve it.)

Games, e.g., Solitaire

Games, e.g., Rush Hour (or: from practice to games to AI models)

Photo made out of Hanna Neumann (between HN, Birch, CSIT, December 2020).

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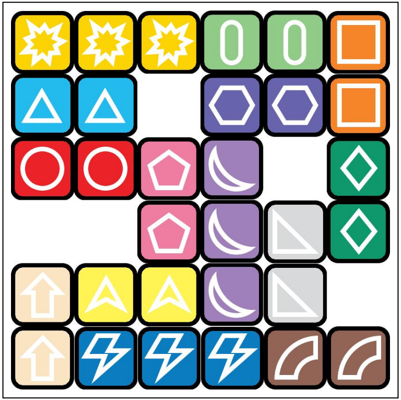

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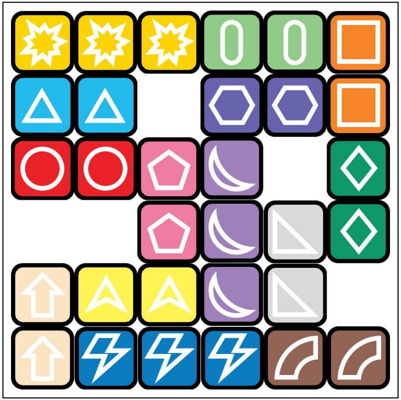
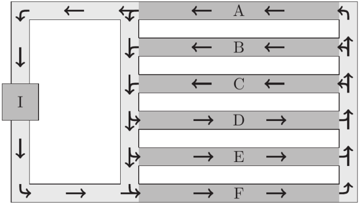


Modeling this, including the automated video creation was (is) a 6 pt. project in S1 2023.

Automated Factories (here: Greenhouse)

- Factory takes imagines of all plants, and decides on their further treatments.
- Factory controls their movements via the conveyor belts.

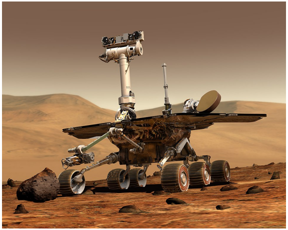


Source: https://www.lemnatec.com/

Further reading:

- Malte Helmert and Hauke Lasinger. "The Scanalyzer Domain: Greenhouse Logistics as a Planning Problem". In: Proceedings of the 20th International Conference on Automated Planning and Scheduling (ICAPS 2010). AAAI Press, 2010, pp. 234-237
- The IPC Scanalizer Domain in PDDL (see paper above).

Robotics (here: Mars Rovers Spirit and Opportunity)



Source: left `https://commons.wikimedia.org/wiki/File:KSC-03PD-0786.jpg`
middle `https://commons.wikimedia.org/wiki/File:Curiosity_Self-Portrait_at_%27Big_Sky%27_Drilling_Site.jpg`
right `https://commons.wikimedia.org/wiki/File:NASA_Mars_Rover.jpg`

Further reading:

- Pascal Bercher and Daniel Höller. "Interview with David E. Smith". In: Künstliche Intelligenz 30.1 (2016). Special Issue on Companion Technologies, pp. 101-105. DOI: 10.1007/s13218-015-0403-y
- `https://www.nasa.gov/` and papers about MAPGEN (for references, see also article above).

Informal Problem Introduction

We consider classical planning problems, which consist of:

- An initial state s_I – all "world properties" true in the beginning.
- A set of available actions – how world states can be changed.
- A goal description g – all properties we'd like to hold.

What do we want?

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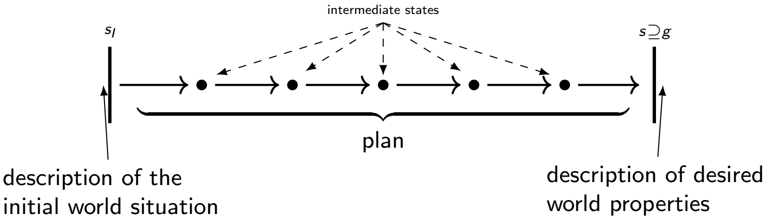
What do we want?

$\rightarrow$ Find a plan that transforms s_I into g.



Problem Definition

A classical (or STRIPS) planning problem $\langle V, A, s_I, g \rangle$ consists of:

- V is a finite set of state variables (also called: facts or propositions).
  - States are collections of state variables.
  - We assume the closed world assumption, i.e., all variables not mentioned in a state s do not hold in that state (in contrast to: it's not known whether they hold or not).
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A. What a solution is!

Problem Definition, cont'd (Solutions)

Action application:

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PLANEX $= \{\langle \mathcal{P} \rangle : \mathcal{P}$ is a classical planning problem $\langle V, A, s_I, g \rangle$ that has a solution.$\}$.

Example Problem

Let $s_I = \{\mathrm{At}_{\mathrm{LivingRoom,R}}, \mathrm{At}_{\mathrm{Garage,Remote}}, \mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{TV}_{\mathrm{Off}}\}$

Rick's actions:

- $\mathrm{PushBox}_R$: $(\{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{LivingRoom,R}}\},\{\mathrm{At}_{\mathrm{LivingRoom,M}}\},\emptyset)$

Meeseeks's actions:

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- $\mathrm{GoToLivingRoom}_R$: $(\{\mathrm{At}_{\mathrm{Garage,R}}\},\{\mathrm{At}_{\mathrm{LivingRoom,R}}\},\{\mathrm{At}_{\mathrm{Garage,R}}\})$
- $\mathrm{PickUpRemote}_R$: $(\{\mathrm{At}_{\mathrm{Garage,R}}, \mathrm{At}_{\mathrm{Garage,Remote}}\},\{\mathrm{Has}_{\mathrm{Remote,R}}\},\{\mathrm{At}_{\mathrm{Garage,Remote}}\})$

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- $\mathrm{TurnTVOn}_R$: $(\{\mathrm{Has}_{\mathrm{Remote,R}}, \mathrm{At}_{\mathrm{LivingRoom,R}}, \mathrm{TV}_{\mathrm{Off}}\},\{\mathrm{TV}_{\mathrm{On}}\},\{\mathrm{TV}_{\mathrm{Off}}\})$

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- $\mathrm{GiveRemote}_M$: $(\{\mathrm{Has}_{\mathrm{Remote,M}}, \mathrm{At}_{\mathrm{LivingRoom,M}}, \mathrm{At}_{\mathrm{LivingRoom,R}}\},\{\mathrm{Has}_{\mathrm{Remote,R}}\},$
  $\{\mathrm{Has}_{\mathrm{Remote,M}}, \mathrm{At}_{\mathrm{LivingRoom,M}}\})$

$g = \{\mathrm{TV}_{\mathrm{On}}\}$

Example Problem, Solutions

Recap: $s_I = \{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{LivingRoom,R}}, \mathrm{At}_{\mathrm{Garage,Remote}}, \mathrm{TV}_{\mathrm{Off}}\}$.

Solution 1 (Rick does it himself):

1. $\mathrm{GoToGarage}_R$: $s_1 = \{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{Garage,R}}, \mathrm{At}_{\mathrm{Garage,Remote}}, \mathrm{TV}_{\mathrm{Off}}\}$
2. $\mathrm{PickUpRemote}_R$: $s_2 = \{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{Garage,R}}, \mathrm{Has}_{\mathrm{Remote,R}}, \mathrm{TV}_{\mathrm{Off}}\}$
3. $\mathrm{GoToLivingRoom}_R$: $s_3 = \{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{LivingRoom,R}}, \mathrm{Has}_{\mathrm{Remote,R}}, \mathrm{TV}_{\mathrm{Off}}\}$
4. $\mathrm{TurnTVOn}_R$: $s_4 = \{\mathrm{At}_{\mathrm{LivingRoom,Box}}, \mathrm{At}_{\mathrm{LivingRoom,R}}, \mathrm{Has}_{\mathrm{Remote,R}}, \mathrm{TV}_{\mathrm{On}}\}$

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Solution 2 (Rick uses a Meeseeks):

1. PushBox_R: $s_1 = \{\text{At}_{\text{LivingRoom,Box}}, \text{At}_{\text{LivingRoom,R}}, \text{At}_{\text{Garage,Remote}}, \text{At}_{\text{LivingRoom,M}}, \text{TV}_{\text{Off}}\}$
2. GoToGarage_M: $s_2 = \{\text{At}_{\text{LivingRoom,BMSox}}, \text{At}_{\text{LivingRoom,R}}, \text{At}_{\text{Garage,Remote}}, \text{At}_{\text{Garage,M}}, \text{TV}_{\text{Off}}\}$
3. PickUpRemote_M: $s_3 = \{\text{At}_{\text{LivingRoom,Box}}, \text{At}_{\text{LivingRoom,R}}, \text{At}_{\text{Garage,M}}, \text{Has}_{\text{Remote,M}}, \text{TV}_{\text{Off}}\}$
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6. TurnTVOn_R: $s_6 = \{\text{At}_{\text{LivingRoom,Box}}, \text{At}_{\text{LivingRoom,R}}, \text{Has}_{\text{Remote,R}}, \text{TV}_{\text{On}}\}$

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Classical Planning is in **PSPACE**

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- Now we can decide $P(s_I, g', 2^{|V|})$ in the usual way, i.e., $P(s_1, s_2, k)$ iff there exists an s, such that $P(s_1, s, {}^{k}\!/_{2})$ and $P(s, s_2, {}^{k}\!/_{2})$.
- Each state is only polynomially large, and we only need to do this split $\log(2^{|V|})$ often. So we only need poly space to do this search.
- Thus, PLANEX $\in$ **PSPACE**.

Classical Planning is **PSPACE**-hard

We reduce from a poly-space-bounded Turing Machine.

- We define $s_I = \{at_{1,q_0}, in_{0,B}, in_{1,w_1}, \ldots in_{|w|,w_{|w|}}, in_{|w|+1,B}, \ldots, in_{pol(|w|)-1,B}\}$ with
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  1. $(\{at_{i,q}, in_{i,x}\}, \{do_{i,q,x}\}, \{at_{i,q}\})$
  2. $(\{do_{i,q,x}, in_{i,x}\}, \{in_{i,y}\}, \{in_{i,x}\})$
  3. $(\{do_{i,q,x}, in_{i,y}\}, \{at_{i+1,q'}\}, \{do_{i,q,x}\})$
  4. don't provide actions for $at_{-1,q}$ and $at_{pol(|w|),q}$ (for any q)

  $\rightarrow$ Uses the new variable(s) $do_{i,q,x}$. Provide the analogous action for left-movement.

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A. So that we can say: Planning is even **PSPACE**-hard if we have only 2 preconditions and 2 effects! Think of 2-SAT vs. 3-SAT! (Here, the 2 precs/effs correspond to the 3!)

Optimal (or: Cost-Bounded) Classical Planning is **PSPACE**-complete

$\text{PLANEX}_k = \{\langle \mathcal{P}, k \rangle : \mathcal{P}$ is a planning problem with a solution $\bar{a}$, $|\bar{a}| \leq k.\}$

Note that the k in the index here is a String, i.e., literally the letter k, not a number.
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  - We know that if a solution exists at all, then one exists up to length $2^{|V|}$. Recap: This is because there is no point in repeating any of the $2^{|V|}$ states.
  - We can thus check for plan existence up to the number $\min(k, 2^{|V|})$.
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- We now show **PSPACE**-hardness:
  - We again exploit that if there exists a plan at all, there is one up to length $2^{|V|}$.
  - We thus reduce from PLANEX: We take an arbitrary problem $\mathcal{P} \in$ PLANEX and create a cost-bounded one by choosing $k = 2^{|V|}$, where V are the variables of $\mathcal{P}$. Note that this construction is polytime because we can encode k using only $\log(k)$ bits.

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- But is every instance **PSPACE**-hard?
  - Clearly not! What about the problem $(\emptyset, \emptyset, \emptyset, \emptyset)$?
  - Think of SAT – which is **NP**-complete. How about 2-*CNF-SAT*?
- So, which factor(s) make planning hard? And what if they were not there?
  - If we identify such a special case in a given instance we could use a more efficient algorithm than one designed for the general case.
  - If we can establish a special case we can solve the easier case and use its solution as approximation to the solution of the actual problem. (E.g., as heuristic in a search.)

Delete-free (or Delete-relaxed) Problems, Definition

Reminder: Classical planning problems have the form (V, A, s_I, g).

- A problem (V, A, s_I, g) is called delete-free if the following holds: for all $(pre, add, del) \in A$ holds: $del = \emptyset$

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- Given a problem $\mathcal{P} = (V, A, s_I, g)$, we call $\mathcal{P}' = (V', A', s_I', g')$ delete-relaxed version of $\mathcal{P}$ if $V = V'$, $s_I' = s_i$, $g' = g$, and $A' = \{(pre, add, \emptyset) : (pre, add, del) \in A\}$.

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- $\text{PLANEX}_{DR} = \{\langle \mathcal{P} \rangle : \mathcal{P}$ is a solvable classical delete-free planning problem.$\}$

Now, what's true?

- PLANEX_{DR} is **PSPACE**-complete (?)
- PLANEX_{DR} is **NP**-complete (?)
- PLANEX_{DR} is in **NP**, not **NP**-hard, and not in **P** (?)
- PLANEX_{DR} is in **P** (?)

Delete-free Planning is in **P**

Observations:

- Applying an action twice is pointless, so we can delete each applied action.

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Algorithm 1: Decision-procedure for delete-free planning.

Data: Set A of delete-free actions, initial state s_I, goal description g
Result: Whether the delete-free problem is solvable

$s \leftarrow s_I$;
repeat
    foreach action $a \in A$ **do**
        if $pre(a) \subseteq s$ **then**
            $s = s \cup add(a)$;
            delete a from A;
until A is not modified;
return $s \supseteq g$;

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Observations:

- Applying an action twice is pointless, so we can delete each applied action.
- Each iteration costs at most $\mathcal{O}(|A|)$ and we can delete at most $|A|$ times.
- Thus, runtime is in $\mathcal{O}(|A|^2)$, so $\text{PLANEX}_{DR} \in \mathbf{P}$.

Cost-bound Delete-Free Planning is in **NP**

$\text{PLANEX}_{k-DR} = \{\langle \mathcal{P}, k \rangle : \mathcal{P}$ is a delete-free planning problem with a solution $\bar{a}$, $|\bar{a}| \leq k.\}$
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- Let $\mathcal{P}$ (delete-free problem) and number k be given.
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- Guess up to k actions and an order among them.
- Return true if sequence is executable and makes goal true. Right?
- No! That's a **NEXPTIME**-procedure! k is encoded binarily...
  Instead, we limit the number of actions that we guess.
- No action has to be executed twice! So we only guess up to $|A|$ (distinct) actions.
- Thus, we perform the above procedure for the number $\min(k, |A|)$.
- This results in an **NP** membership procedure/proof.

Cost-bound Delete-Free Planning is **NP**-hard

We show that PLANEX$_{k-DR}$ is **NP**-hard.

- We reduce from CNF-SAT.
- Let $\varphi = \underbrace{\{C_1, \ldots, C_n\}}_{\text{clauses}}$, $C_j = \underbrace{\{\varphi_{j_1}, \ldots, \varphi_{j_k}\}}_{\text{literals}}$, and $V = \underbrace{\{x_1, \ldots, x_m\}}_{\text{variables}}$.

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- Let $\varphi = \underbrace{\{C_1, \ldots, C_n\}}_{\text{clauses}}$, $C_j = \underbrace{\{\varphi_{j_1}, \ldots, \varphi_{j_k}\}}_{\text{literals}}$, and $V = \underbrace{\{x_1, \ldots, x_m\}}_{\text{variables}}$.
- For each boolean variable $x_i \in V$ add two actions to A:

$x_i \mapsto \top$	$x_i - \top$, $x_i - set$
$x_i \mapsto \bot$	$x_i - \bot$, $x_i - set$

Cost-bound Delete-Free Planning is **NP**-hard

We show that PLANEX_{k-DR} is **NP**-hard.

- We reduce from CNF-SAT.
- Let $\varphi = \underbrace{\{C_1, \ldots, C_n\}}_{\text{clauses}}$, $C_j = \underbrace{\{\varphi_{j_1}, \ldots, \varphi_{j_k}\}}_{\text{literals}}$, and $V = \underbrace{\{x_1, \ldots, x_m\}}_{\text{variables}}$.
- For each boolean variable $x_i \in V$ add two actions to A:

$$\boxed{x_i \mapsto \top} \; x_i - \top,\; x_i - set \qquad \boxed{x_i \mapsto \bot} \; x_i - \bot,\; x_i - set$$

- For each positive $\varphi_{j_i} = x_{j_i}$ or negative $\varphi_{j_i} = \neg x_{j_i}$ add

$$x_{j_i} - \top \; \boxed{\begin{array}{c}\text{“}x_{j_i} = \top\text{”} \\ C_j \mapsto \top\end{array}} \; C_j - \top \quad \text{or} \quad x_{j_i} - \bot \; \boxed{\begin{array}{c}\text{“}x_{j_i} = \bot\text{”} \\ C_j \mapsto \top\end{array}} \; C_j - \top$$

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- We reduce from CNF-SAT.
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- For each boolean variable $x_i \in V$ add two actions to A:

$\boxed{x_i \mapsto \top}$ effects: $x_i-\top$, x_i-set

$\boxed{x_i \mapsto \bot}$ effects: $x_i-\bot$, x_i-set

- For each <u>positive</u> $\varphi_{j_i} = x_{j_i}$ or <u>negative</u> $\varphi_{j_i} = \neg x_{j_i}$ add

$x_{j_i}-\top$ $\boxed{\begin{array}{c}\text{“}x_{j_i} = \top\text{”} \\ C_j \mapsto \top\end{array}}$ $C_j-\top$ or $x_{j_i}-\bot$ $\boxed{\begin{array}{c}\text{“}x_{j_i} = \bot\text{”} \\ C_j \mapsto \top\end{array}}$ $C_j-\top$

- $g = \{x_i-set \mid 1 \le i \le m\} \cup \{C_j-\top \mid 1 \le j \le n\}$
- φ is satisfiable if and only if a plan of size $n + m$ exists.

Cost-bound Delete-Free Planning is **NP**-hard

We show that PLANEX_{k-DR} is **NP**-hard.

- We reduce from CNF-SAT.
- Let $\varphi = \underbrace{\{C_1, \ldots, C_n\}}_{\text{clauses}}$, $C_j = \underbrace{\{\varphi_{j_1}, \ldots, \varphi_{j_k}\}}_{\text{literals}}$, and $V = \underbrace{\{x_1, \ldots, x_m\}}_{\text{variables}}$.
- For each boolean variable $x_i \in V$ add two actions to A:

  $\boxed{x_i \mapsto \top}$ with effects $x_i - \top$, $x_i - set$

  $\boxed{x_i \mapsto \bot}$ with effects $x_i - \bot$, $x_i - set$

- For each positive $\varphi_{j_i} = x_{j_i}$ or negative $\varphi_{j_i} = \neg x_{j_i}$ add

  $x_{j_i} - \top$ $\boxed{\begin{array}{c}\text{“}x_{j_i} = \top\text{”}\\ C_j \mapsto \top\end{array}}$ $C_j - \top$ or $x_{j_i} - \bot$ $\boxed{\begin{array}{c}\text{“}x_{j_i} = \bot\text{”}\\ C_j \mapsto \top\end{array}}$ $C_j - \top$

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- φ is satisfiable if and only if a plan of size $n + m$ exists.

You are not done yet! Don't forget to show this is a reduction!