COMP3630 / COMP6363

week 12: **Examples from Hierarchical Planning**

All taken from literature

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The Australian National University

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Content of this Chapter

Hierarchical Task Network (HTN) Planning:

- Examples
- Formal Problem Definition(s)
- Expressivity Analysis
- Complexity Analysis

Examples

Terminology and some Background

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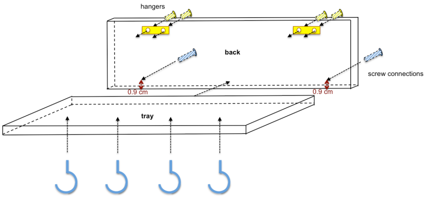
- Why defining/solving a hierarchical problem?
  - As above: In many real-world applications, knowledge is given in form of control rules: we know the steps required to perform some task.
  - More control on the generated plans, since all the "rules" need to be obeyed. We can exclude (more) undesired plans! (Exactly how formal grammars do!)
  - Plans can be presented more abstract by relying on task hierarchies.
  - We can solve/express more complex problems! (Spoiler)

Example: Do-It-Yourself (DIY) Assistant, The Task

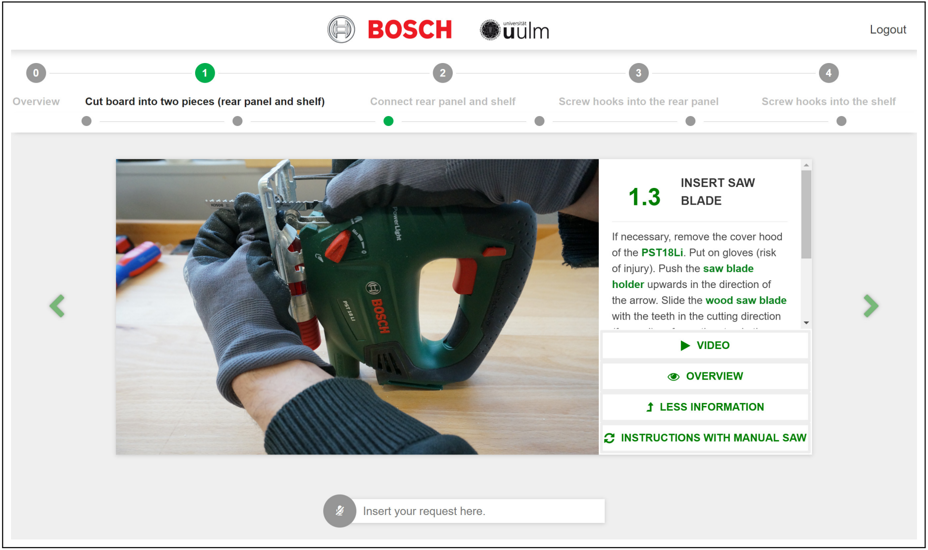


The material:

- Boards (need to be cut first)
- Electrical devices like drills and saws
- Attachments like drill bits and materials like nails

Further reading: Pascal Bercher et al. "Do It Yourself, but Not Alone: Companion-Technology for Home Improvement – Bringing a Planning-Based Interactive DIY Assistant to Life." Künstliche Intelligenz – Special Issue on NLP and Semantics, 35: 367–375. 2021.

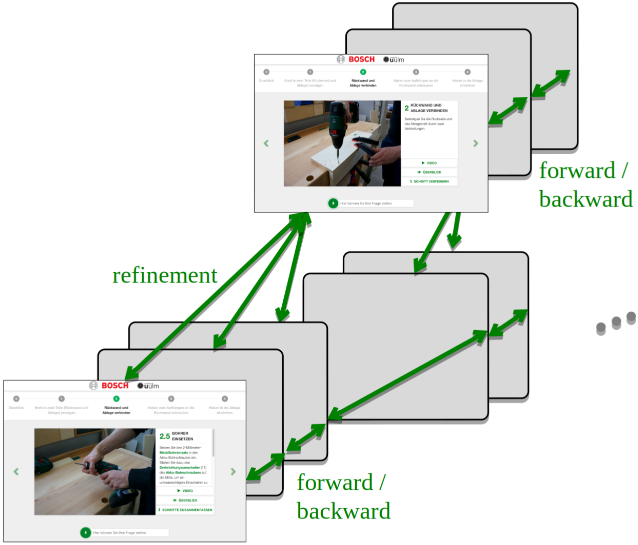
Example: Do-It-Yourself (DIY) Assistant, User Interface



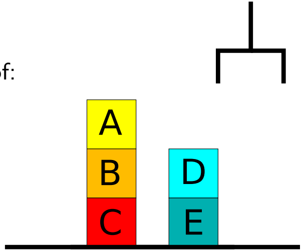
Example: Do-It-Yourself (DIY) Assistant, Task Hierarchy

Abstract Level

Detailed Level



Recap: Blocksworld via Classical Planning

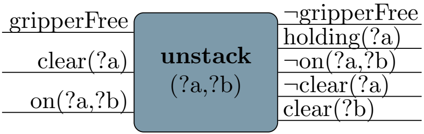


We consider classical planning problems, which consist of:

- All existing state variables V.
- An initial state $s_I \in 2^V$.
- A set of available actions A.
- A goal description $g \subseteq V$.

$\rightarrow$ Find an action sequence (i.e., a plan) that transforms s_I into a state $s \supseteq g$.

For example, one of the available actions is:



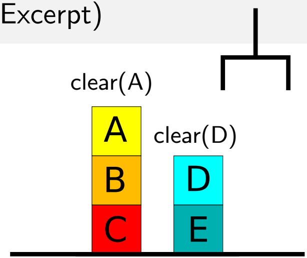
- For an action to be executable, all preconditions must hold.
- Actions change states by adding or deleting their effects.

Blocksworld via HTN Planning (and HDDL Excerpt)

makeClear($?b$)

done
pre: clear($?b$)

(empty)



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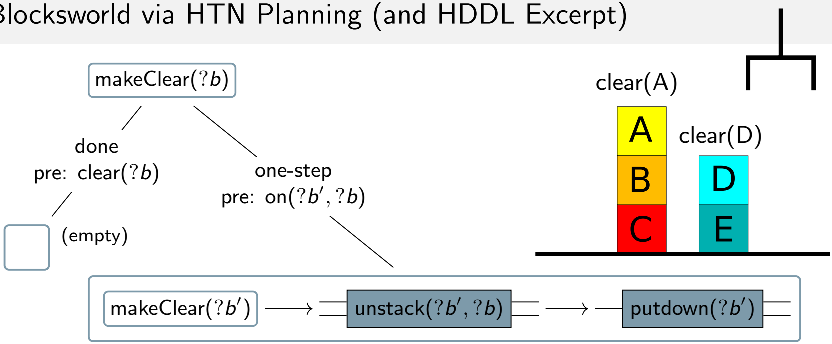


```
(:task makeClear :parameters (?b - block))

(:method one-step
 :parameters (?b1 ?b2 - block)
 :task (makeClear ?b1)
 :precondition (and (on ?b2 ?b1))
 :ordered-tasks (and (makeClear ?b2)
                     (unstack ?b2 ?b1)
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```

makeClear(C)

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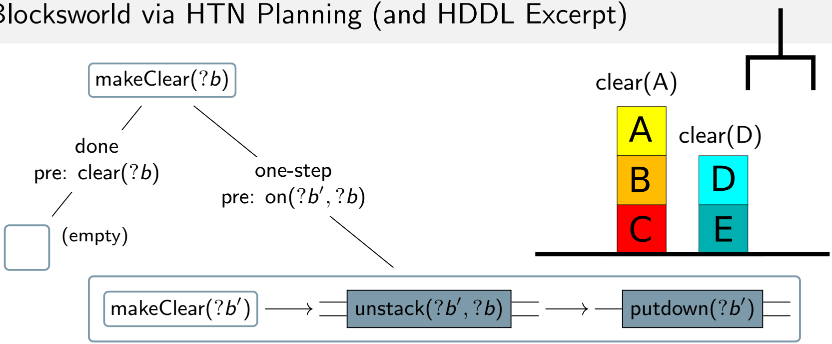


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makeClear(B)
unstack(B,C)
putdown(B)
~~makeClear(C)~~

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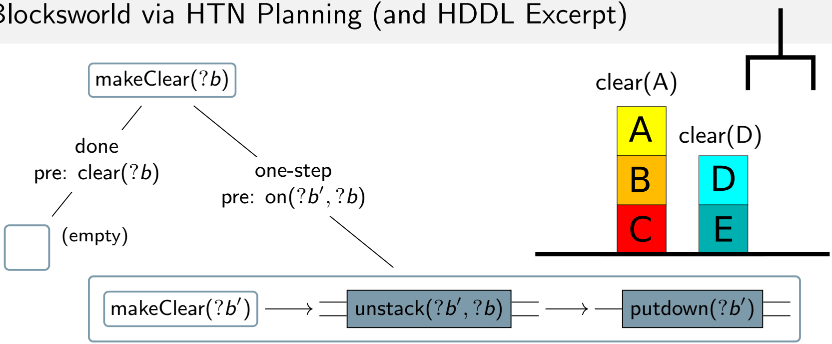


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makeClear(A)
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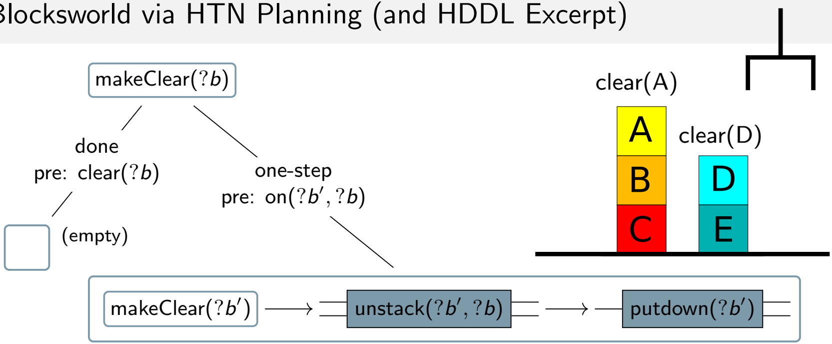


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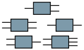
Formalism

Introduction to HTN Planning

primitive
tasks



compound
tasks

$\mathcal{P} = (V, P, \delta, C, M, s_I, c_I, g)$

- V a set of facts
- P a set of primitive task names
- $\delta : P \to (2^V)^3$ the task name mapping
- C a set of compound task names

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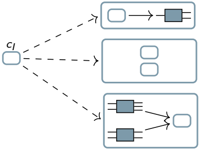

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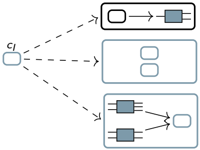

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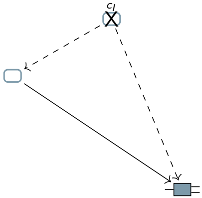

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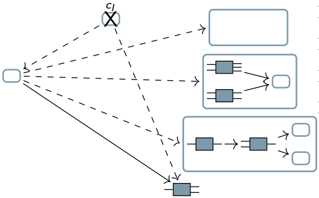

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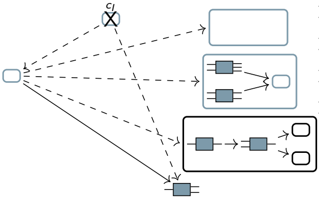

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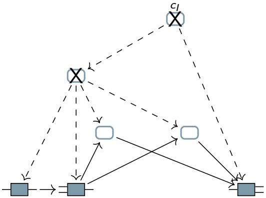

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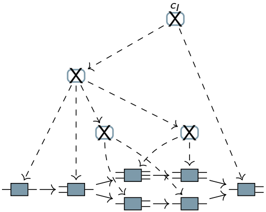

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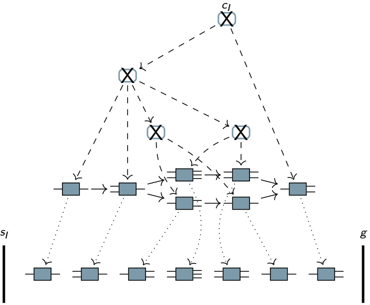

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- $s_I \in 2^V$ the initial state
- $g \subseteq V$ the (optional) goal description

We must find a task network *tn*, such that:

- it is a refinement of c_I,
- only contains primitive tasks, and
- has an executable linearization that makes the goals in g true.

Decomposition, formally

- Task networks are tuples $(T, \prec, \alpha)$ consisting of a set of task IDs (labels) T and a strict partial order $\prec \subseteq T \times T$. I.e., $\prec$ is irreflexive and transitive (and hence asymmetric). $\alpha : T \to P \cup C$ maps task IDs to the actual tasks (i.e., their names).

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- Note that this definition becomes trivial if all methods are totally ordered. It then perfectly coincides with the definition of using a production rule.

HTN Planning: Solution Criteria in more Detail

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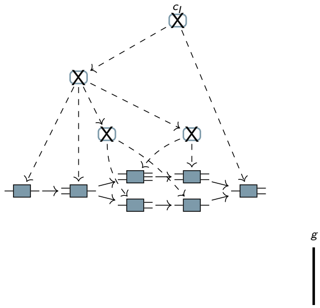

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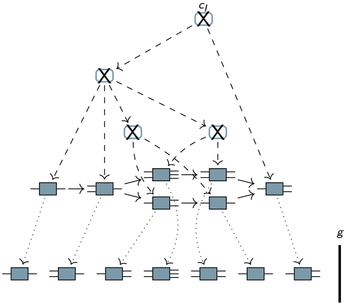

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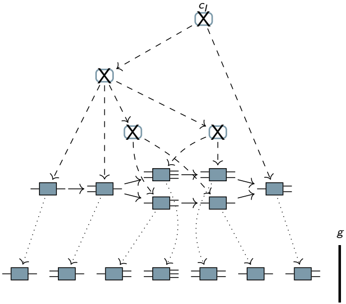

An action sequence is called executable if every action is executable in its state:

- Let $s \in 2^V$ be a state, $p \in P$, and $\delta(p)$ an action with $\delta(p) = (pre, add, del)$ and $pre, add, del \subseteq V$.
- Then, p is executable in s iff $pre \subseteq s$.
- Then, p executed in s leads to new state $s' = (s \setminus del) \cup add$.

Alternative Definition of HTN Planning

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- Thus, any solution set $sol(\mathcal{P})$ is a language. Let:
  - $L_H(\mathcal{P}) = \{\bar{p} \mid \bar{p} \in sol(\mathcal{P}')$, where $\mathcal{P}'$ ignores all facts $\}$
  - $L_C(\mathcal{P}) = \{\bar{p} \mid \bar{p} \in sol(\mathcal{P}')$, where $\mathcal{P}'$ is the induced classical problem $\}$
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  - L_H just looks at the words produced by the hierarchy, (ignores executability)
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$\to$ Thus: $sol(\mathcal{P}) = L_H(\mathcal{P}) \cap L_C(\mathcal{P})$.

This observation gives a new/simplified view on HTN planning:

HTN planning = classical planning + grammar to filter solutions

!! Maybe the most important interpretation of knowledge about HTN Planning **!!**

Expressivity Analysis

Recap: Chomsky Hierarchy, extended

We can define the following Language classes:

- $\mathcal{A}ll = \{L \mid L \text{ is a language}\}$
- $\mathcal{CSL} = \{L \mid L \text{ is a context-sensitive language}\}$
- $\mathcal{CF} = \{L \mid L \text{ is a context-free language}\}$
- $\mathcal{R}eg = \{L \mid L \text{ is a regular language}\}$
- $\mathcal{CLASSIC} = \{L(\mathcal{P}) \mid \mathcal{P} \text{ is a (propositional) classical planning problem.}\}$

We know that $\mathcal{CLASSIC} \subsetneq \mathcal{R}eg \subsetneq \mathcal{CF} \subsetneq \mathcal{CSL} \subsetneq \mathcal{A}ll$.

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Now, we also have:

- $\mathcal{HTN} = \{L(\mathcal{P}) \mid \mathcal{P} \text{ is an HTN planning problem.}\}$
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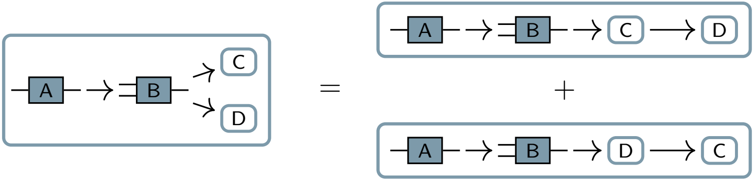
- It is known that the intersection of a context-free and regular language is context-free. (We didn't prove that yet, but the idea is a product automaton of PDA and DFA.) $\square$

On the (Non?)equivalence of PO and TO HTN models

Each partially ordered task network is just a compact representation of its linearizations:



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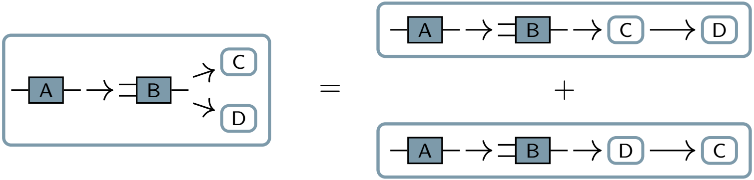

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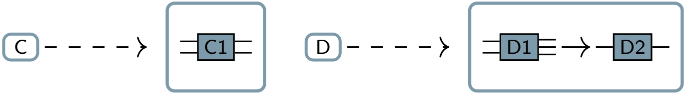

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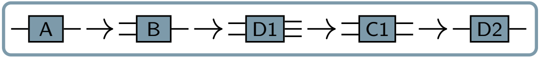

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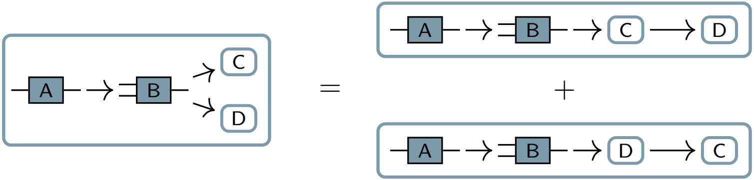

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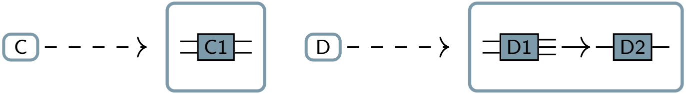

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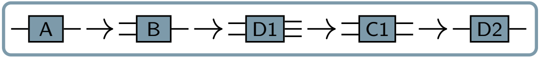

No! Not anymore...

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Note: There are more special cases that were considered in literature.

Complexities

Complexity of HTN Planning (General Case)

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- Our desired primitive task network tn contains only one executable linearization $\omega = \omega_1, \omega_2, \ldots, \omega_{2n-1}, \omega_{2n}$:
  - $\omega^1 = \omega_1, \omega_3, \ldots, \omega_{2n-1}$, $|\omega^1| = n$, and $\omega^1 \in L(G)$
  - $\omega^2 = \omega_2, \omega_4, \ldots, \omega_{2n}$, $|\omega^2| = |\omega^1|$, and $\omega^2 \in L(G')$
- Encoding given in next slide!

□

Reduction, Shown by Example

Let $G = (\overbrace{N = \{H, Q\}}^{\text{non-terminals}}, \overbrace{\Sigma = \{a, b\}}^{\text{terminals}}, \overbrace{R}^{\text{rules}}, \overbrace{H}^{\text{start symbol}})$
and $G' = (N' = \{D, F\}, \Sigma' = \{a, b\}, R', D)$.

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$M = M(G) \cup M(G')$ (translated production rules of G and G')

$$tn_I = (\underbrace{\{t, t'\}}_{T}, \underbrace{\emptyset}_{\prec}, \underbrace{\{t \mapsto H, t' \mapsto D\}}_{\alpha})$$

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Hardness:

- Reduction from a polyspace-bounded ATM.
  (We know **AP** = **PSPACE**, but it also holds: **APSPACE** = **EXPTIME**)
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Conclusion

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- Recall one interesting result from week 10: Delete-relaxed HTN planning is **NP**-complete, although even the shortest solution may be exponential.

Conclusion of the Course

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  - the languages from the Chomsky Hierarchy (and the Pumping lemmas)
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  - the languages from the Chomsky Hierarchy (and the Pumping lemmas)
  - the classes $\mathcal{R}$, $\mathcal{RE}$, non-$\mathcal{RE}$
- The last weeks we were investigating the "computational complexity" of various languages. Noteworthy are:
  - The investigated complexity classes and their relationship. (Which are known?)
  - The difference between **NP** and **co-NP**.
  - The difference of membership, hardness, completeness – and reductions.

Final Remarks

- Don't forget that:
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- Keep monitoring the forum! Read all questions/answers !!
- I hope you enjoyed the course!
- Good luck in the exam! (And your other exams.)

Thank you for taking this course!

A **special Thank you** to those who attended in person! :)