Change the World – How Hard Can That Be?
On the Complexity of Fixing Planning Models

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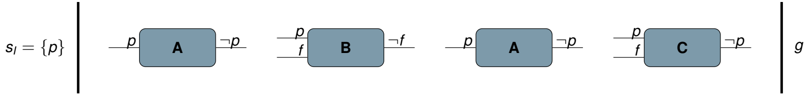
Introduction

Motivation



An infeasible plan in which the action A deletes the fact p that is required by the actions A, B and C, and the environment does not have the fact f that is required by B and C as well.

- Counter-factual Explanations. (How to make the plan executable?)

	A	**B**	**C**
p	delete $\neg p$	N/A	N/A
	N/A	delete p	delete p
f	add f	N/A	N/A
	N/A	delete f	delete f

- Modeling assistance.

Scenario

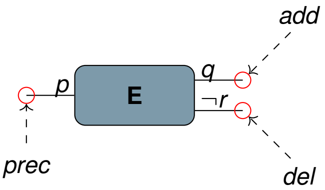
Given a plan that is supposed to be a solution to a planning problem, but it is actually not, we want to change the planning model so that it can be.

- Considering the problem in the context of hierarchical (HTN) & non-hierarchical planning.
- Complexity Study.

Changing Planning Models in Classical Planning

Basic Terminologies

- A state s is a set of proposition variables
- An action is a tuple $(prec, add, del)$
  - Preconditions $prec$: $prec \subseteq s$.
  - Effects add & del: $(s \setminus del) \cup add$.



An example of the action $\mathbf{E} = (\underbrace{\{p\}}_{prec}, \underbrace{\{q\}}_{add}, \underbrace{\{r\}}_{del})$.

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Given an action sequence, we want to change actions' preconditions and effects to make the sequence executable.

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- FIX-ADD: Adding a variable to an action's add list.
- FIX-DEL: Removing a variable from an action's delete list.

Problem Definition: Definition

Definition (Fix-Action^k_X, $X \subseteq \{\textsc{Prec}, \textsc{Add}, \textsc{Del}\}$ and $|X| \geq 1$)

Given an action sequence $\overline{a}$, is there a way to make $\overline{a}$ executable by using the respective changes according to the value of X at most k times.

- If $\textsc{Prec} \in X$, Fix-Prec is allowed.
- If $\textsc{Add} \in X$, Fix-Add is allowed.
- If $\textsc{Del} \in X$, Fix-Del is allowed.

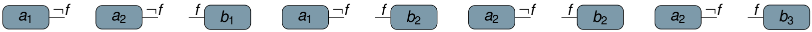
Results: FIX-PREC & FIX-DEL

Theorem

FIX-ACTION$_{\text{PREC,DEL}}$ *is in P.*

Proof.

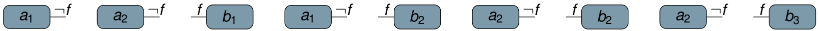
Reducing to the vertex cover problem with a bipartite graph as the input. □



An example of input action sequence in which $a_1 = a_2 = (\emptyset, \emptyset, \{f\})$ and $b_1 = b_2 = b_3 = (\{f\}, \emptyset, \emptyset)$.

*More complicated cases where $prec \cap add \cap del \neq \emptyset$ for some actions are considered in the proof presented in our paper.

Results: Reducing to the Vertex Covering Problem



Definition (Vertex Cover)

Given a graph, finding the minimum subset of the vertices such that for every edge, at least one of its endpoints is in the set.

The minimal vertex cover is $\{a_1, a_2\}$. $\Longrightarrow$
The optimal change is removing f from the actions a_1 and a_2.

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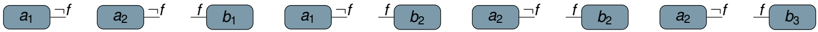

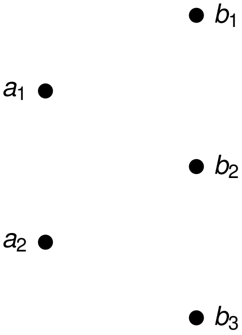

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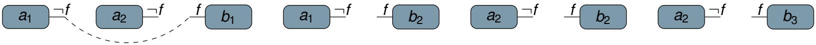

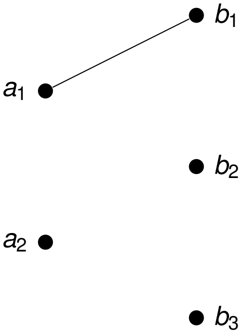

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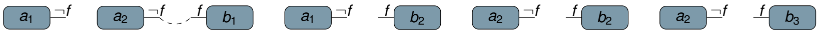

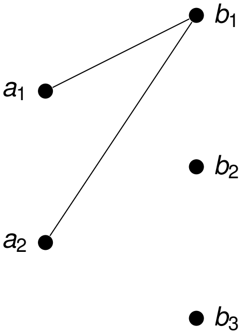

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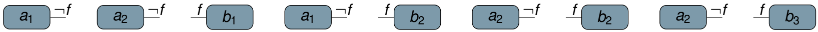

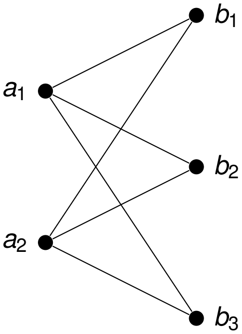

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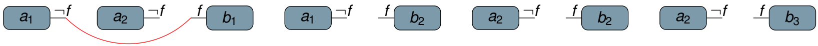

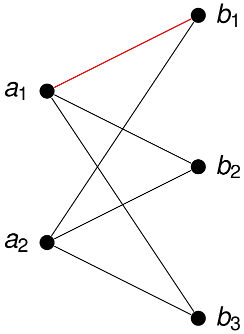

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Results: Only FIX-ADD

Theorem

FIX-ACTION$^k_{\text{ADD}}$ *is NP-complete.*

Proof.

Suppose the given action sequence is $\overline{a} = a_1 \cdots a_n$.

- Membership: Guessing at most $\min\{k, \sum_{i=1}^{n} |prec(a_i)|\}$ changes.
- Hardness: Reducing from the set covering problem.

□

Results: Set Covering

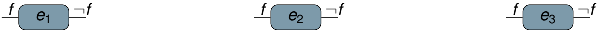
Definition

Given a set of sets $\mathcal{S} = \{S_1, \cdots, S_n\}$ and an integer k, is there a subset S' of S such that $|S| \leq k$ and $\bigcup_{S \in \mathcal{S}} S = \bigcup_{i=1}^{n} S_i$.

E.g., $\mathcal{S} = \{\underbrace{\{e_1\}}_{S_1}, \underbrace{\{e_1, e_2\}}_{S_2}, \underbrace{\{e_3\}}_{S_3}\}$ and $k = 2$.

- A solution to this instance is $S' = \{S_2,\ S_3\}$.

Results: Reduction from the Set Covering Problem



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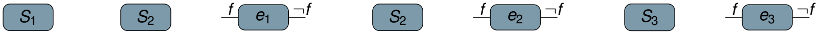

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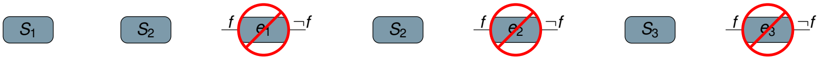

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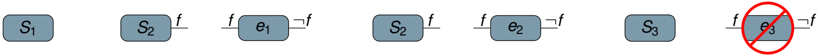

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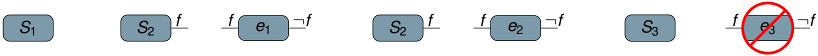

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Corollary

FIX-ACTION$_X^k$ *is NP-complete if* ADD $\in X$.

Changing Planning Models in HTN Planning

HTN Planning: Components

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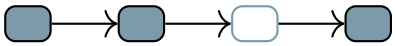
- Task networks 

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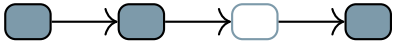
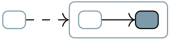
- Task networks 
- Methods 

HTN Planning: Formalism

tn_I

$P = (D, tn_I, s_I)$ with
$D = (F, N_c, N_p, \delta, M)$

- F: A set of proposition variables called facts.
- N_c: A set of compound tasks.
- N_p: A set of primitive tasks (actions).
- δ: $N_p \rightarrow 2^F \times 2^F \times 2^F$
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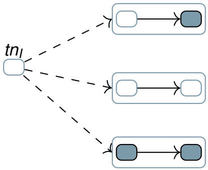

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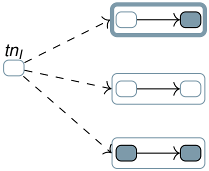

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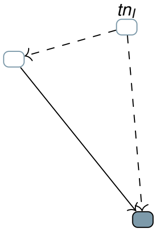

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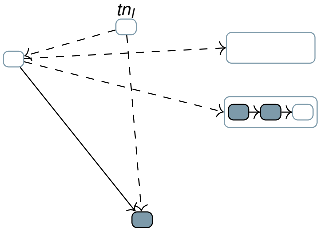

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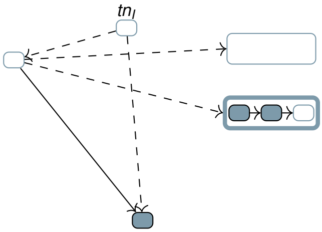

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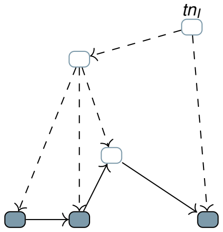

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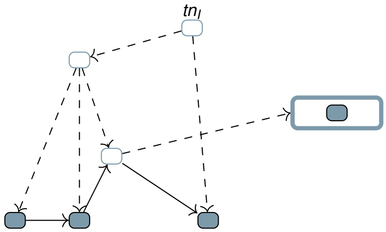

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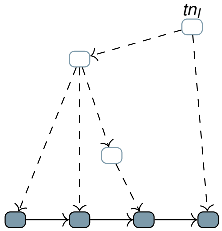

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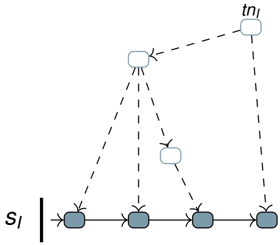

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Changing Methods: Allowed Changes

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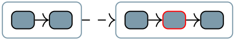

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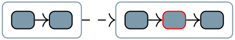
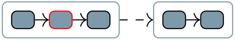


- DEL-ACTION: Removing an action from a method.



Given Only an Action Sequence: Problem Definition

Definition (Fix-Meths$_X$, $X \subseteq \{\textsc{Add}, \textsc{Del}\}$ and $|X| \geq 1$)

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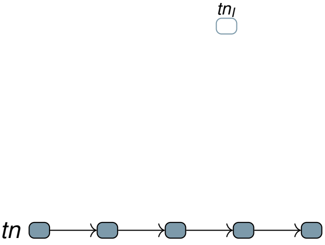

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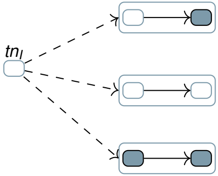

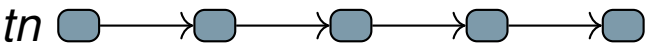

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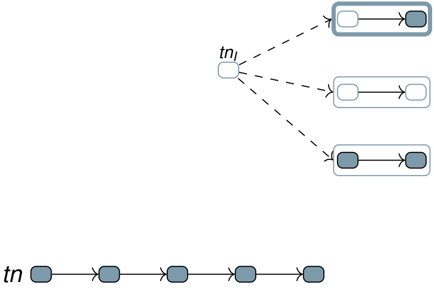
Given an HTN planning problem P and an action sequence tn, is there a way to change the methods in P by applying the respective changes according to the value of X so that tn becomes a solution.



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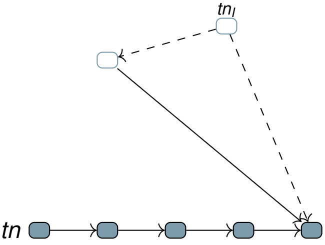

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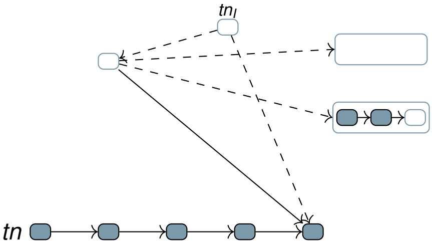

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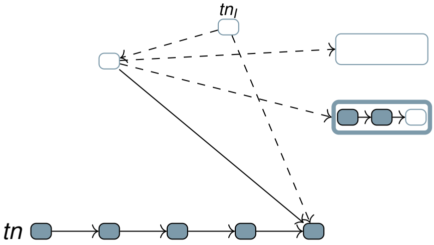

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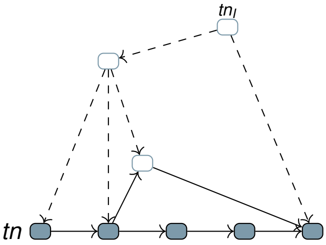

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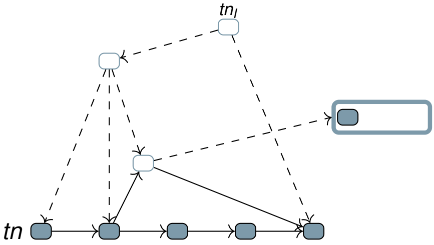

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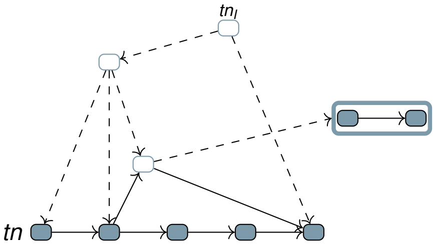

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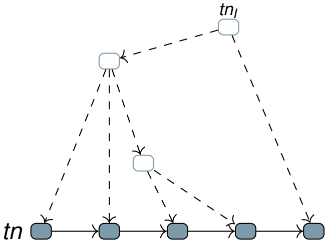

Given Only an Action Sequence: Membership

Theorem

FIX-METHS$_X$ *is in NP.*

Proof.

Key Observation: If there exists a sequence of changes that turns *tn* into a solution, then there must be one of length bounded by a polynomial.

- A totally-ordered HTN planning can be regarded as a CFG.
- Parsing based plan verification algorithms.

□

Given Only an Action Sequence

Theorem

FIX-METHS$_X$ *is NP-hard.*

Proof.

- Reducing from the independent set problem.

□

Given an Action Sequence & a Method Sequence: Problem Definition

Definition (FIX-SEQ$_X$)

Given a planning problem P, a task network tn, and a method sequence $\overline{m}$. Is there a way to change the methods in P by using the allowed changes specified by X (e.g., ADD and DEL) such that $\overline{m}$ decomposes the initial task network of P into tn.

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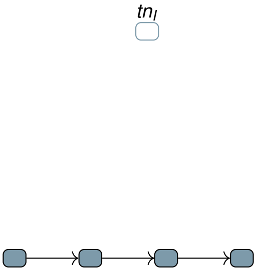

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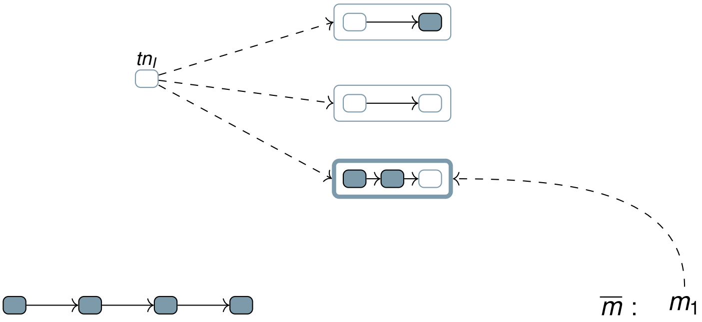

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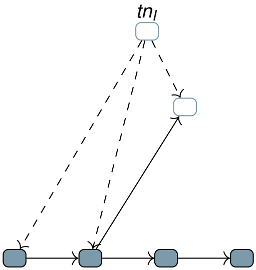

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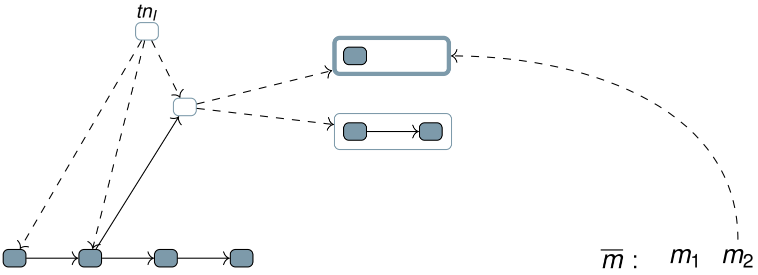

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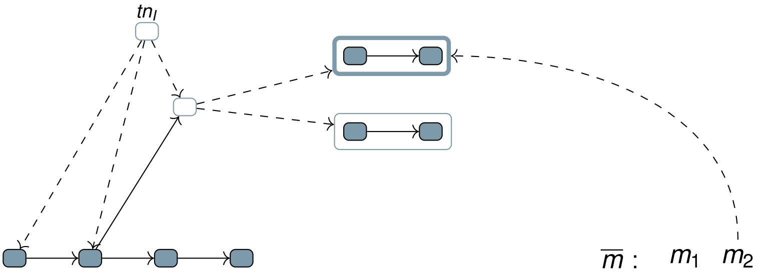

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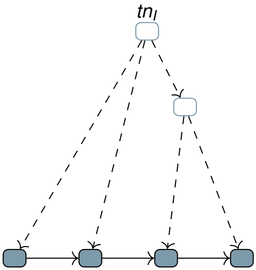

$\overline{m}$: m_1 m_2

Given an Action Sequence & a Method Sequence

Theorem

FIX-SEQ$_X$ *is NP-complete.*

Proof.

- Reduction from the independent set problem again.

□

Optimization: Finding the Minimal Number of Changes

Definition ($\textsc{Fix-Meths}_X^k$ & $\textsc{Fix-Seq}_X^k$)

Given an integer k, the problems $\textsc{Fix-Meths}_X^k$ and $\textsc{Fix-Seq}_X^k$ are identical to $\textsc{Fix-Meths}_X$ and $\textsc{Fix-Seq}_X$ except that we bounded the number of changes by k.

Corollary

$\textsc{Fix-Meths}_X^k$ and $\textsc{Fix-Seq}_X^k$ are NP-complete.

Conclusion

Summary

Changes	Complexity
prec *del* *prec*, *del*	P
add *prec*, *add* *del*, *add* *prec*, *add*, *del*	NP-complete

Computational Complexity of Changing Actions.

Methods Given?	Changes	Complexity	
		Any Changes	*k Changes*
No	Del Add Add, Del	NP-complete	NP-complete
Yes	All	NP-complete	NP-complete
Yes: Unique	All	P	P

Computational Complexity of Changing Methods.