Eliminating Redundant Actions in Partially Ordered Plans

A Complexity Analysis

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Motivation

Try to remove redundant actions in partially ordered plans.

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Find some solution fast
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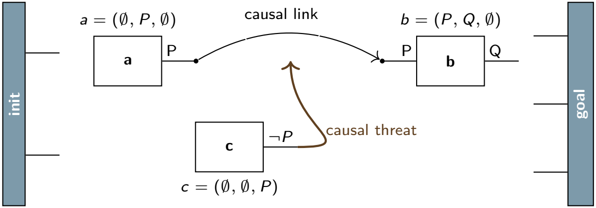
- Find some solution fast and post-optimize afterwards
- Mixed-initiative planning
- Plan-reuse
- Preprocessing may improve explanation of necessity of actions

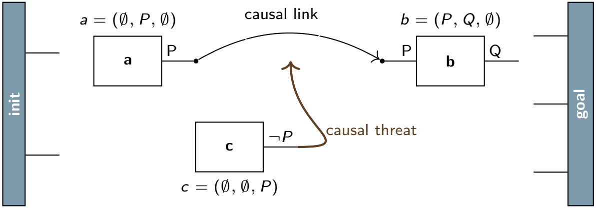
Partial Plan



Partial Plan

A *partial* plan is a tuple $P = (PS, \prec, CL)$, where

- PS is a finite set of *plan steps* $ps = (l, a)$ with l being a label unique in PS and $a \in A$ an action,
- $\prec$ is a partial order on PS, and
- CL is a finite set of causal links.



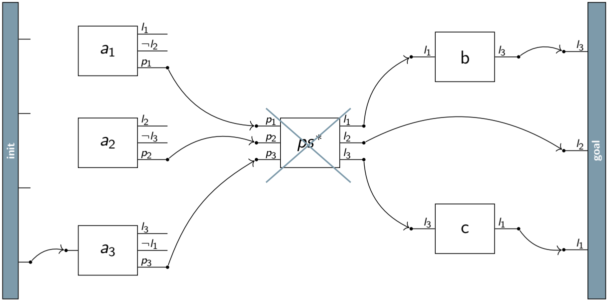
POCL Plan

$P = (PS, \prec, CL)$ is a *partial-order causal link* (POCL) solution plan iff

- all preconditions are supported by causal links and
- there are no causal threats.

$\Rightarrow$ All linearizations are classical solution plans.

The Problem Remove & Repair



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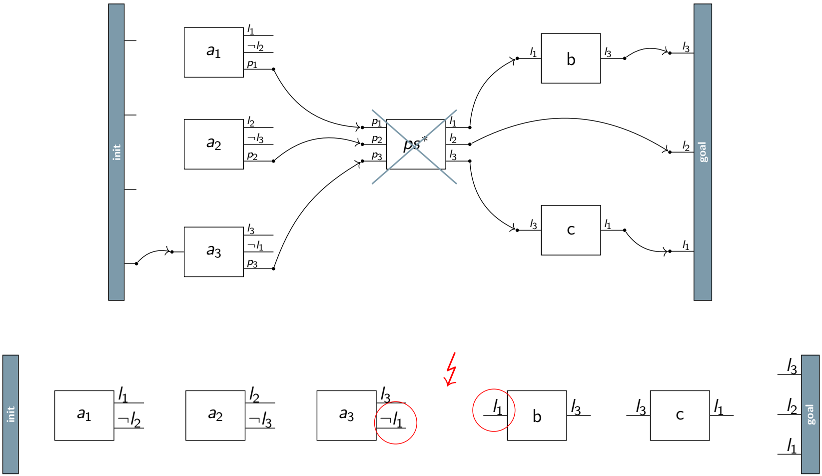

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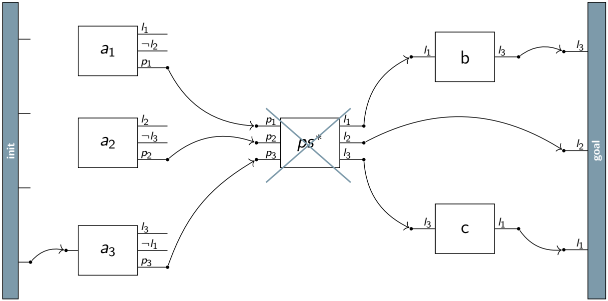

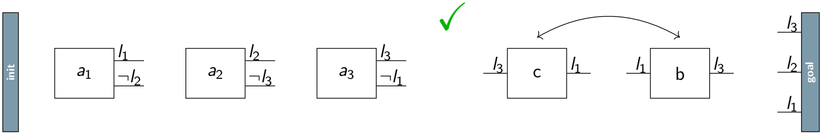

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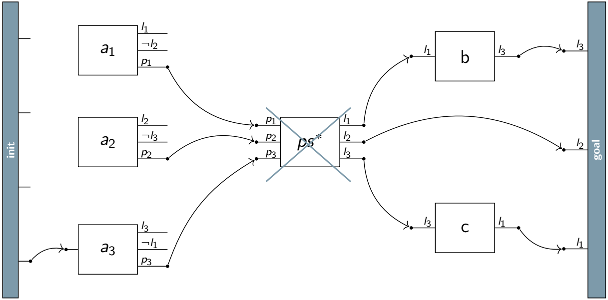
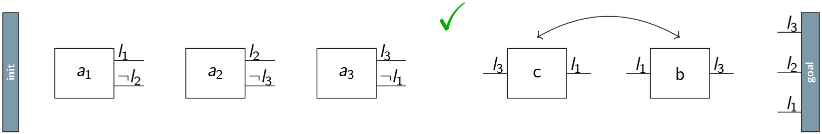




→ Not every linearization is still a solution but some are!

The Problem Remove & Repair

Definition (Remove & Repair (R&R))

Given

- a POCL plan P for some problem Π and
- one plan step that will be removed.

Decision problem:

- Is there an ordering-refinement of P (only *adding* causal links and ordering constraints is allowed) without this plan step that is still a solution for Π?

The Problem Remove & Repair

Theorem

$\mathrm{R\&R}$ *is NP-complete.*

The Problem Remove & Repair

Theorem

R&R *is NP-complete.*

Proof.

1. Membership ✓
2. Cycle dissolving Pairs (CdP) is NP-complete.
3. Reduction from CdP

□

Cycle dissolving Pairs (CdP)

Definition (Cycle dissolving Pairs (CdP))

Given

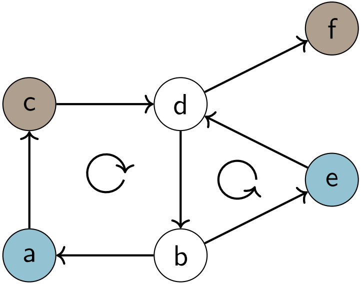
- a directed graph G and
- a partition of a subset of its vertex set, such that each element has size two.

Decision problem:

- Is it possible to make G acyclic by deleting at most one vertex of each partition element?

Example 1

$$\widehat{V} = \{\{c, f\}, \{a, e\}\}$$



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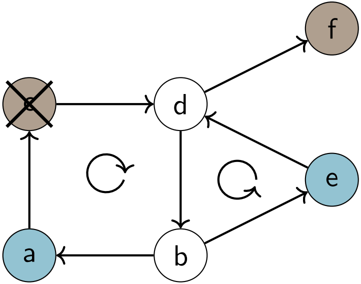

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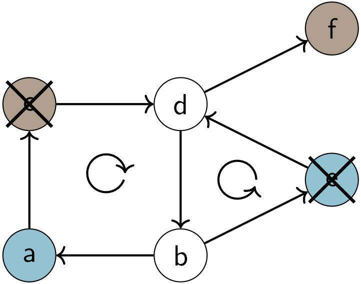

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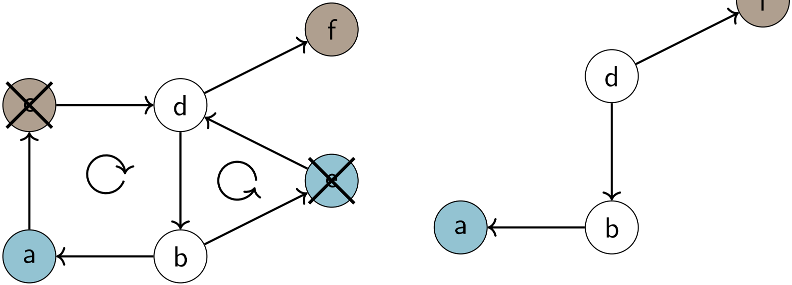

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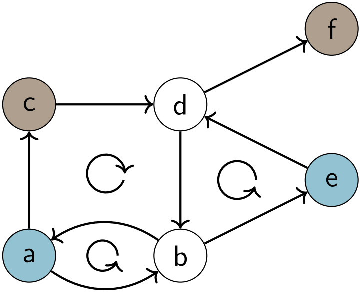

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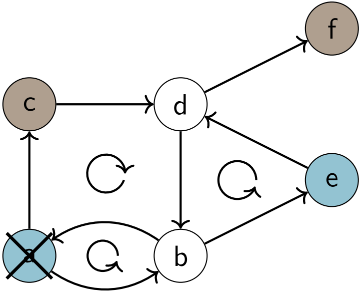

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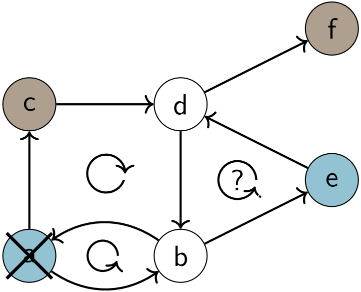
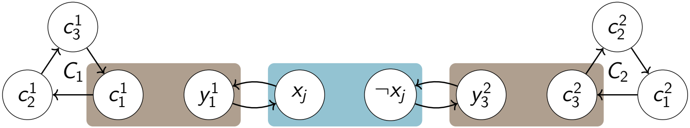


Proof Idea: CDP is NP-complete

Reduction from 3-SAT

$$\phi = \underbrace{(x_j, \neg x_b, \neg x_c)}_{C_1} \wedge \underbrace{(\neg x_d, x_e, \neg x_j)}_{C_2} \wedge \cdots \wedge \underbrace{(\ldots)}_{C_n}$$



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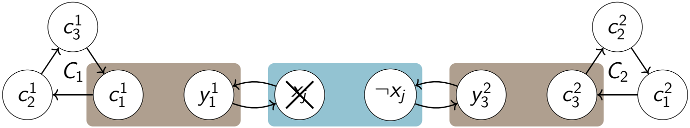

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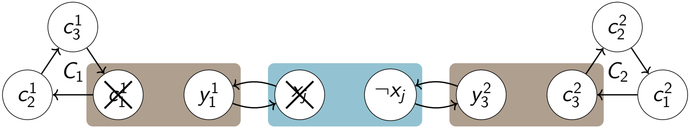

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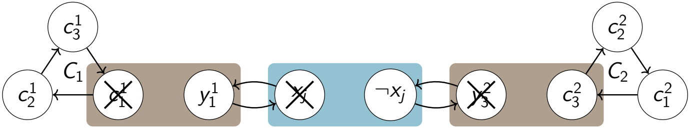

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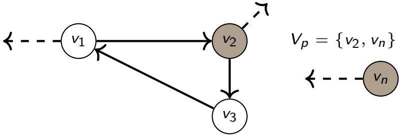

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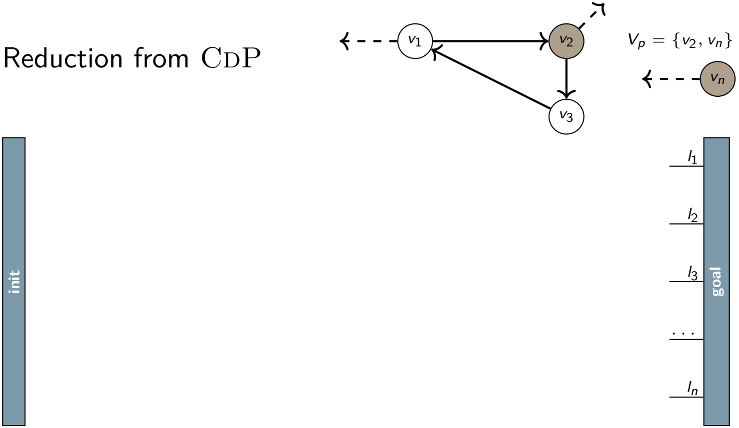

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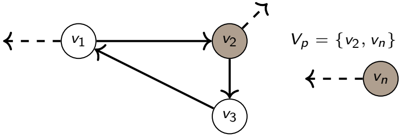

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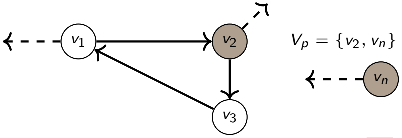

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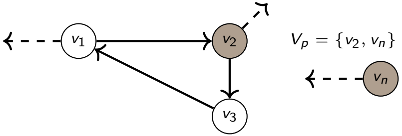

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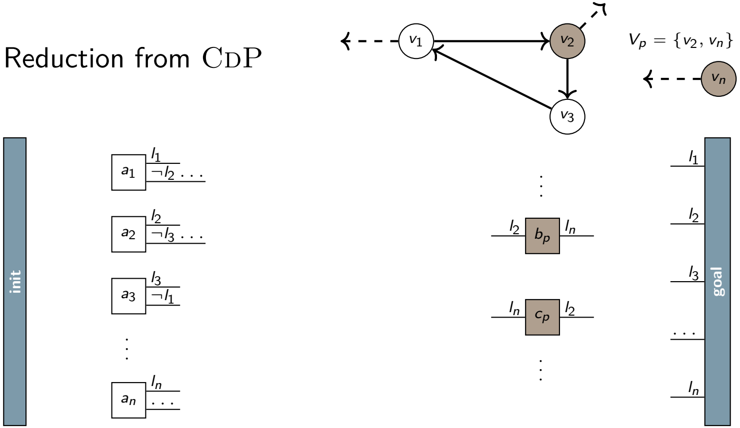

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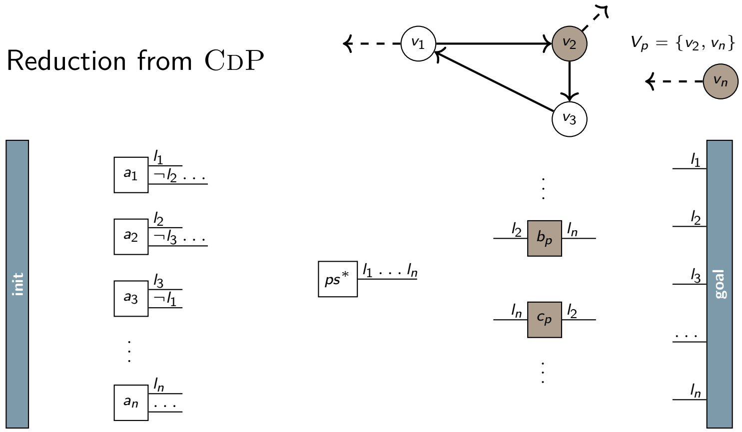

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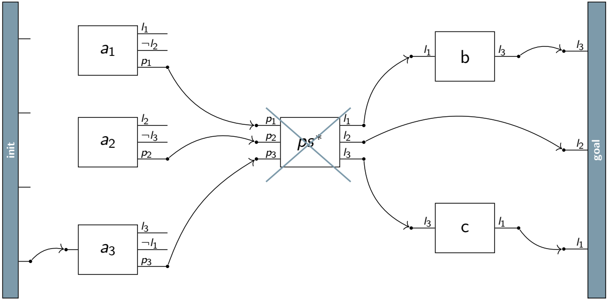

Parameterized Complexity

Theorem

$\#_{\mathit{Atweens}}$ - $\mathrm{R\&R}$ *is fixed-parameter tractable*

Summary of Main Results

plan	actions to be removed			
	one *given*	$\exists$ one	k given	$\exists k$

Table: Computational complexity of problems concerned with the removal of actions.

[1] [Fink and Yang, 1992, Nakhost and Müller, 2010]

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Cycle dissolving Pairs (CdP)

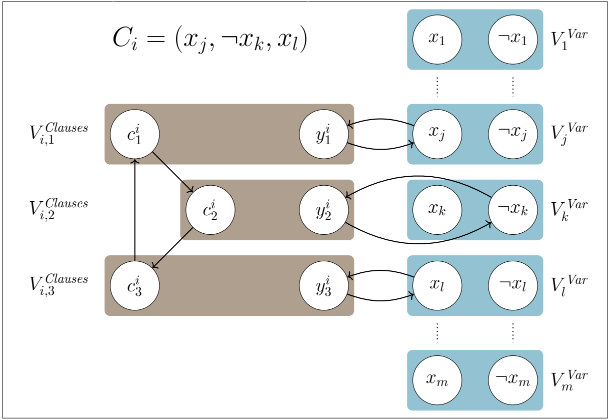
The decision problem Cycle dissolving Pairs (CdP) is defined as follows:

Definition (CdP)

Let $G = (V, E)$ be a directed graph and $\widehat{V} = \{V_1, V_2, \ldots, V_m\}$ a partition of a subset of V such that $|V_i| = 2$ for all $1 \leq i, j \leq m$, $i \neq j$. Is there a $U \subseteq V$ such that

- $U \subseteq \bigcup_{V_i \in \widehat{V}} V_i$,
- $|U \cap V_i| \leq 1$ for all $i = 1 \ldots m$ and
- $G \setminus U$ is acyclic?

Proof CdP NP-complete



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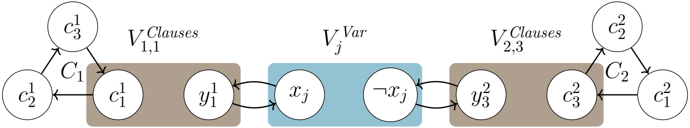

Parameterized Complexity

Parameter of the $\mathrm{R\&R}$ instance: $\#_{Atweens}$

The number of plan steps satisfying all of the following three properties:

- They are ordered (not necessarily directly) behind the removed plan step,
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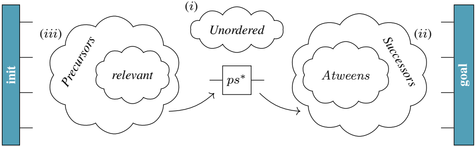

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Nakhost, H. and Müller, M. (2010).
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