## **Tight Bounds for Hybrid Planning**

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# Introduction





Introduction

In this paper we study:

- A hierarchical approach to planning, more specifically:
  - An extension of Hierarchical Task Network (HTN) planning
  - Hybrid planning: HTN planning with causal links
- Specifically, we investigate the computational complexity of
  - The *plan existence problem* = Does there exist a solution?
  - The *plan verification problem* = Does plan *P* solve the problem?





#### Introduction to Actions and Causal Links



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compound tasks



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  - P a set of primitive task names
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Hybrid Planning

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A solution task network tn must:

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- $s_I \in 2^F$  the initial state
- $g \subseteq F$  the (optional) goal description

- be a refinement of c<sub>l</sub>,
- only contain primitive tasks, and
- have an executable linearization.



#### Hybrid Planning Problem Example



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- decomposition methods specify to which subtasks causal links get inherited down



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# **Complexity Results**





## Main messages:

Causal links don't increase computational complexity

- We took all membership proofs from HTN planning and showed how to adapt them to still work despite the inheritance of causal links (no entirely new proofs were required)
- Various complexity classes are covered: (N)P, (N)EXPTIME, PSPACE, EXPSPACE – all the same as in HTN planning
- Plan verification is already NP-complete in the absence of a task hierarchy (i.e., if the initial plan is already primitive)

For the actual proofs check out our paper or see you at the poster Thank you for listening!

